

Effect of velocity ratio on plane mixing layer development: Influence of the splitter plate wake

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Abstract. An experimental study has been conducted to investigate the effect of velocity ratio on the approach of a plane mixing layer to self-similarity. Plane mixing layers with five different velocity ratios (0.5, 0.6, 0.7, 0.8 and 0.9) were generated in a newly designed mixing layer wind tunnel with both initial boundary layers tripped. For each velocity ratio, mean flow and turbulence measurements were obtained at eight streamwise locations with a single cross-wire probe. The results indicate that the splitter plate wake plays a very dominant and, in some cases, a lasting role in the development of the mixing layer. For velocity ratios between 0.5 and 0.7, self-similarity of the mixing layer was observed with the asymptotic states comparable. Mixing layers with the higher velocity ratios failed to achieve a self-similar state within the measurement domain, although a slow approach to it was apparent. The development distance decreased with increasing velocity ratio up to 0.7, after which it appeared to increase. Almost all of the observed effects may be attributed to the presence of the splitter plate wake and its complex interaction with the mixing layer.

List of symbols

C_f	boundary layer skin friction coefficient
H	boundary layer shape factor
r	velocity ratio of the two streams, $(=U_2/U_1)$
Re_L	Reynolds number, $(=UL/\nu)$
R	correlation coefficient in least squares fit
U, V, W	mean velocity in the X, Y, Z directions, respectively
U^*	velocity parameter, $[(U - U_2)/(U_1 - U_2)]$
U_0	velocity difference, $(=U_1 - U_2)$
U_e	free-stream velocity in the wind tunnel
u', v', w'	fluctuating velocity components in the X, Y, Z directions, respectively
u, v, w	instantaneous velocity in the X, Y, Z directions, respectively, e.g. $u = U + u'$
X_0	virtual origin of the mixing layer
X, Y, Z	cartesian coordinates for streamwise, normal, and spanwise directions, respectively
Y_0	centerline of mixing layer from error function fit
δ	mixing layer width from error function fit
δ_{99}	initial boundary layer thickness
ζ	similarity coordinate $[(Y - Y_0)/\delta]$
Θ	initial boundary layer momentum thickness
λ	modified velocity ratio $[(1 - r)/(1 + r)]$
λ_n	initial instability wavelength in the mixing layer

σ	spreading parameter $[=1/(d\delta/dX)]$
σ_0	spreading parameter for single-stream mixing layer
$\bar{}$	(overbar) Time-averaged quantity
$()_{\max}$	maximum value at given X -station
$()_{\min}$	minimum value at given X -station
$()_1$	value for high-speed side
$()_2$	value for low-speed side

1 Introduction

Mixing layer studies have been popular over the years, not only because of their important role in practical aerodynamics, but also because their asymptotic behavior is thought to be quite simple in theory. Townsend (1976) showed that the governing equations and boundary conditions for the plane turbulent mixing layer can yield "self-similar" solutions for sufficiently high Reynolds number and downstream distance. The necessary conditions for self-preservation are that the mixing layer grows linearly and that the shapes of the mean velocity and turbulence profiles are independent of downstream distance when scaled by the velocity difference and local mixing layer thickness. It is widely believed that mixing layers achieve such a self-similar condition after a sufficient development distance. Attention has, therefore, been directed toward determining the asymptotic spreading rate and turbulence properties of the self-similar mixing layer.

However, many areas of confusion remain regarding even the self-similar properties of mixing layers (Rodi 1975). Mixing layers seem to be very sensitive to small changes in the initial and operating conditions (see Mehta and Westphal 1986, for a review). Amongst the more important parameters that are known to affect mixing layer behavior are the state and thickness of the initial boundary layer and the free-stream turbulence intensity. For two-stream mixing layers, another obvious parameter to consider is the velocity ratio. As a result, poor agreement exists regarding spreading rates and values of turbulence quantities in the self-preserving region which are independent of initial conditions (Rodi 1975). In spite of the theoretical arguments of Townsend

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(1976), it is not entirely clear if the measured mixing layer turbulence structure is indeed independent of velocity ratio. In addition, there is very limited information on how the distance required to achieve a self-similar state is affected by velocity ratio. Note that while single-stream mixing layers are generally found to achieve a self-similar state in a distance equivalent to $1,000 \Theta$ (Bradshaw 1966), no such obvious criteria have been confirmed for the two-stream layer (Mehta and Westphal 1986).

Amongst the earliest works where the question of velocity ratio effects was addressed directly was that due to Spencer (1970). Spencer investigated the development of two-stream mixing layers originating from untripped boundary layers and obtained detailed measurements for two velocity ratios ($r=0.3$ and 0.6). The peak asymptotic Reynolds stress levels (normalized by the velocity difference) in the self-similar region were found to be higher for the lower velocity ratio. Based partly on observations of the fluctuating velocity signals in the mixing layer and partly on turbulence stress measurements, Spencer concluded that the development distance increased with increasing velocity ratio. Yule (1971a) reached a similar conclusion for his mixing layers with almost the same velocity ratios, but with tripped initial boundary layers. He attributed the longer development distance required for the higher velocity ratio case to the higher convection velocities and the larger deficit in the velocity distribution (caused by the wake) which has to be filled by entrainment. However, in contrast to Spencer's data, Yule's results showed an increase in the peak Reynolds stress levels with increasing velocity ratio. This trend was attributed to changes in the "structure of the turbulence" and Yule (1971b) proposed a formulation for the maximum turbulent kinetic energy as a function of velocity ratio. Saiy and Peerless (1978) also noted an increase in the peak Reynolds stress levels in the self-similar regions with increasing velocity ratio. They suggested that the free-stream turbulence, which was relatively high in their case ($u'/U_e \sim 0.6\%$), had an increasingly stronger effect on the mixing layer as the velocity ratio was increased and the layer became weaker. A similar variation of the peak Reynolds stresses with velocity ratio was obtained by Pui and Gartshore (1979) in their wind tunnel with $u'/U_e \sim 1\%$. For one velocity ratio ($r=0.83$), they showed that the spreading rate and peak Reynolds stresses were reduced significantly, to levels comparable with "the lowest realistic values" reported by other researchers, when a new experimental arrangement with a lower turbulence level ($u'/U_e \sim 0.2\%$) was employed. In contrast to all these investigations, Oster and Wygnanski (1982) in their facility, also with $u'/U_e \sim 0.2\%$, found that the maximum streamwise intensity in the self-similar region of mixing layers with velocity ratios of 0.3 , 0.4 , 0.5 and 0.6 was approximately constant. It turns out that the turbulence level in Yule's (1971a) facility was also relatively high ($u'/U_e \sim 1\%$), which explains his observations regarding the increase in peak Reynolds stresses with velocity ratio. The decrease reported by Spencer (1970), however, is not easily explained, especially

since the reported turbulence level in his wind tunnel was only $u'/U_e \sim 0.1\%$.

More recently, Mehta and Westphal (1986) compared the near-field turbulence structure of a single-stream mixing layer to that of a two-stream mixing layer for both tripped and untripped initial boundary layers. In all cases, it was found that the two-stream layer developed to the self-preserving state in a distance much shorter than the single-stream layer, which followed accepted criteria for the development distance. The asymptotic levels of the turbulence quantities in the two-stream layer and the development of the single-stream layer showed agreement with existing data. These results implied that the development distance of mixing layers would *decrease* with *increasing* velocity ratio.

The present study was inspired by this lack of consensus regarding the effects of velocity ratio on the mixing layer properties. There is also a distinct lack of detailed measurements in the near-field, developing region of two-stream mixing layers. The main objective of the present study was to investigate the effect of velocity ratio on mixing layer development and self-similar properties while keeping, as far as possible, all the other parameters unchanged. For this reason, the flow speed, and hence the initial boundary layer properties, of the high-speed stream was kept constant. The initial boundary layers were tripped since significant spanwise variations in the mixing layer structure were found in a recent study where the initial boundary layers were left in a laminar state (Bell and Mehta 1989). The spanwise variations were caused by the appearance of streamwise vortices in the transition region of the geometrically two-dimensional mixing layer. For the present study, it was expected that these spanwise variations would be reduced by tripping the initial boundary layers. While this objective appeared to have been met, the tripped boundary layers exacerbated the effects due to the splitter plate wake.

2 Experimental apparatus and techniques

The experiments were conducted in a newly designed Mixing Layer Wind Tunnel, consisting of two individually driven legs (Fig. 1). The two streams are allowed to merge at the sharp edge of a tapered splitter plate which divides the contraction into equal sections. The included angle at the splitter plate edge, which lies 15 cm within the test section, is about 1° . The test section is 36 cm in the cross-stream direction, 91 cm in the spanwise direction and 366 cm in length. One side-wall is flexible so that it can be adjusted for streamwise pressure gradient control, although for the present measurements, this wall was kept fixed at a constant distance from the opposite wall.

For the present experiments, the leg driven by the bigger blower was operated at a free-stream velocity in the test section of 21 m/s whereas the flow speed in the other leg was varied between 10.5 and 18.9 m/s, thus giving mixing layers with velocity ratios, $r=0.5$, 0.6 , 0.7 , 0.8 and 0.9 . At $r=0.5$,

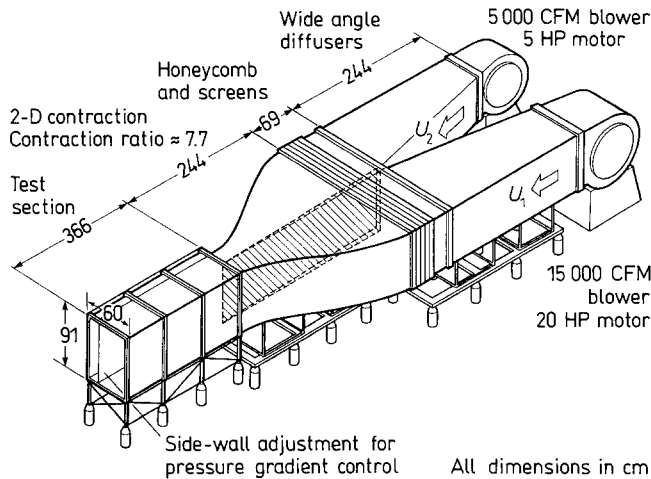


Fig. 1. Schematic of mixing layer wind tunnel

Table 1. Boundary layer properties

Side	U_e (m/s)	δ_{99} (cm)	Θ (cm)	Re_θ	H	$C_f \times 10^3$
High-speed	21.0	0.86	0.096	1,301	1.49	3.99
Low-speed	10.5	0.87	0.098	686	1.53	4.85
Low-speed	12.6	0.85	0.094	781	1.51	4.73
Low-speed	14.7	0.86	0.093	907	1.47	4.63
Low-speed	16.8	0.84	0.092	989	1.47	4.49
Low-speed	18.9	0.83	0.089	1,114	1.45	4.45

this gave a maximum Re_x of 2.75×10^6 , based on the mixing layer convection velocity. At these operating conditions, the measured streamwise turbulence levels (u'/U_e) were about 0.15% and the transverse levels (v'/U_e and w'/U_e) were about 0.05%. The mean core-flow was found to be uniform to within 0.5% and cross-flow angles were less than 0.25° . The boundary layers on the splitter plate were tripped using a 2.5 cm wide roughness strip comprised of a random distribution of round glass beads, about 0.75 mm diameter on the high-speed side and 1 mm diameter on the low-speed side. The selection of this type of three-dimensional trip was based on two primary requirements – the effectiveness of the trip over a relatively large velocity range and a quick recovery of the boundary layer from the perturbation introduced by the trip. Details of the turbulent boundary layers obtained at the splitter plate trailing edge are summarized above in Table 1. The spanwise variation in skin friction coefficient, measured as an indicator for boundary layer two-dimensionality, was found to be less than $\pm 5\%$ about the mean on both sides of the splitter plate.

The measurements were made using a cross-wire probe held on a 3-D traverse and linked to a fully automated data acquisition and reduction system controlled by a MicroVax II computer. The cross-wire probe had $5 \mu\text{m}$ tungsten sensing elements about 1 mm long and positioned about 1 mm apart. The probe was calibrated statically in the potential

core of the flow assuming a 'cosine-law' response to yaw, with the effective angle determined by calibration. The analog signals were filtered (low pass at 30 KHz), DC offset, and amplified (X 10) before being fed into a computer interface containing a fast sample-and-hold A/D converter with 12 bit resolution. Individual statistics were averaged over 5,000 samples obtained at a rate of 400 samples per second.

Data were obtained in the uv -plane with a cross-wire probe at eight streamwise stations within the test section between $X = 10$ to 270 cm. The data were typically measured as profiles along the tunnel centerline although spanwise uniformity checks were also made at selected stations; for example, the standard deviation of the peak primary shear stress was about 4%.

3 Results and preliminary discussion

Following Townsend (1976), the normalizing velocity scale is chosen as the velocity difference across the layer (U_0), and the shear layer thickness (δ), is used to normalize the Y -coordinate. The shear layer thickness and centerline (Y_0) are determined using a least-squares fit of the mean data to the error function profile shape:

$$U^* = (1 + \operatorname{erf}(\zeta))/2 \quad (1)$$

Mean streamwise velocity profiles for all five velocity ratios are plotted in similarity coordinates in Fig. 2. Note that in these and subsequent profile plots showing results for the individual velocity ratios, the axes limits and scales are not always the same. The main feature apparent in these results is the presence of a velocity defect at the upstream stations on the low-speed side of the mixing layer. The velocity defect, which is caused by the presence of the splitter plate wake, increases with velocity ratio, as does the streamwise distance over which it is still discernible. Also shown in these figures is Townsend's (1976) error function relation (Eq. 1). For all velocity ratios, the mean velocity profiles seem to collapse quite well onto the error function profile, once the splitter plate wake is "washed-out" – for $r = 0.5$ to 0.7, the mean profiles were found to fit the error function to within 1% rms error within the range $0.1 < U^* < 0.9$. The rms error was found to increase for the higher velocity ratios, reaching 2.5% at the last station for $r = 0.9$. The splitter plate wake strength is illustrated quantitatively in Fig. 3, where the streamwise development of the maximum normalized velocity defect $[(U_{\min} - U_2)/U_0]$ is plotted. For velocity ratios, $r = 0.5$ to 0.8, the velocity defect is almost completely eliminated by $X \sim 125$ cm. However, the $r = 0.9$ case does not appear to have recovered from the wake effects even by the last measurement station at $X = 267$ cm. The decay of the velocity defect for $r = 0.9$ is much faster than the $1/X^{0.5}$ correlation obtained for a plane wake (Weygandt and Mehta 1989). This is not too surprising since the presence of the superimposed mixing layer obviously leads to more entrainment, and hence, a faster decay of the velocity defect.

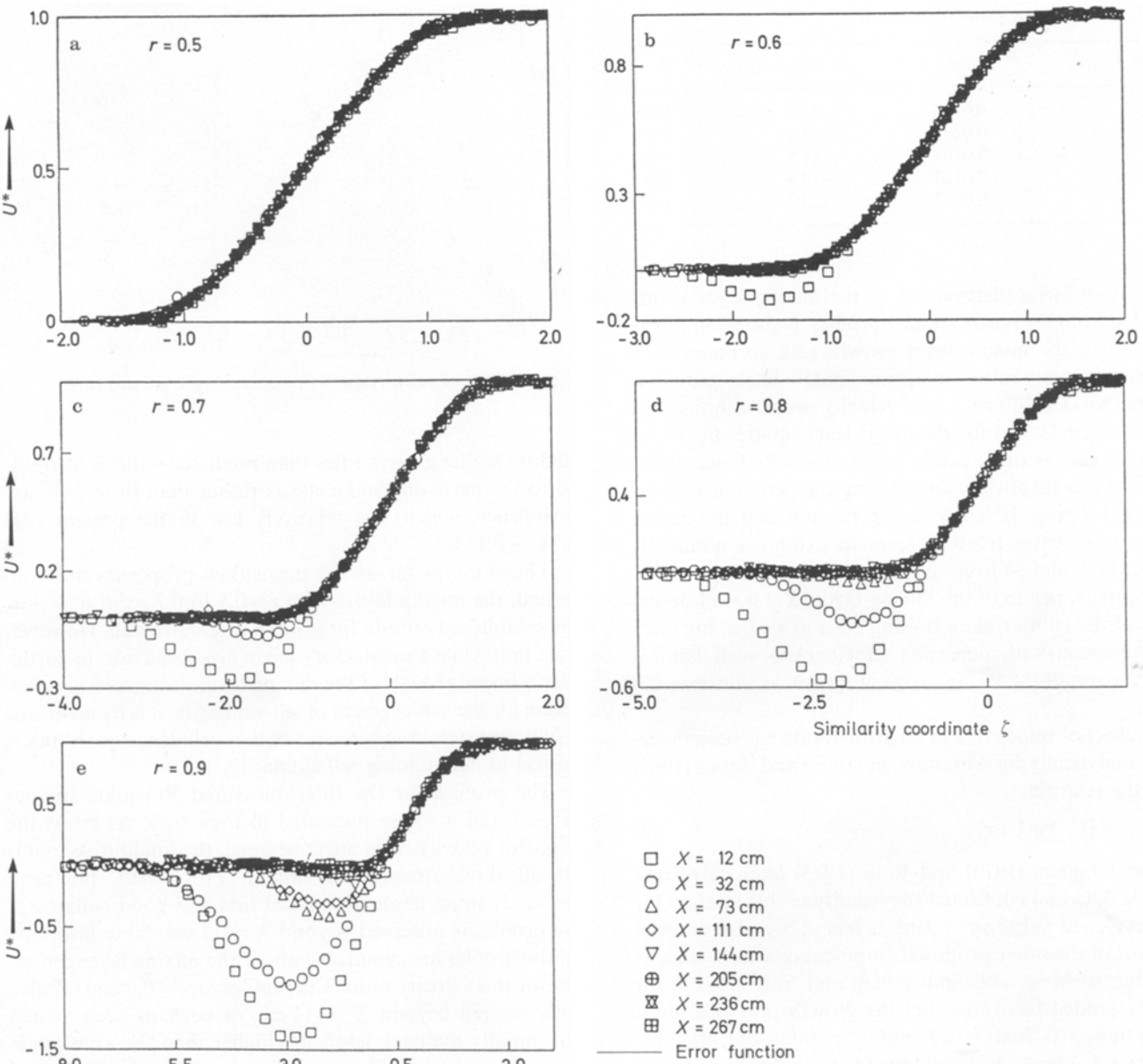


Fig. 2. Mean streamwise velocity profiles. (Note varying scales)

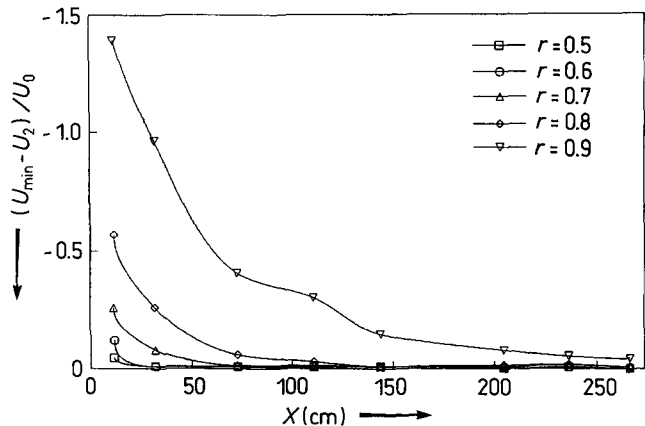


Fig. 3. Streamwise distribution of mean velocity defect

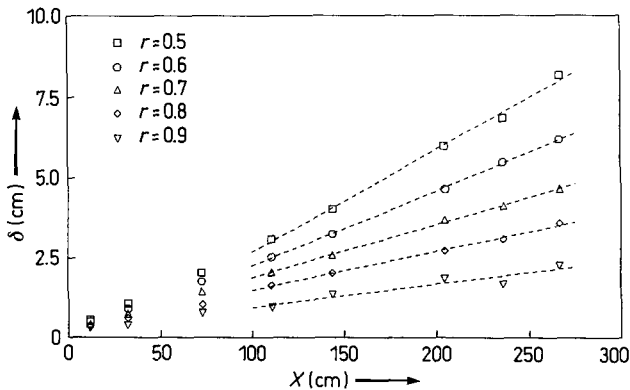


Fig. 4. Two-stream mixing layer growth

Table 2. Mixing layer growth parameters

Velocity ratio	$d\delta/dx$	X_0 (cm)	R
0.5	0.0318	15.7	0.99
0.6	0.0235	4.1	0.99
0.7	0.0168	-11.5	0.99
0.8	0.0124	-19.4	0.99
0.9	0.0073	-28.3	0.87

The streamwise distribution of the mixing layer width, evaluated from the mean velocity profiles, is shown in Fig. 4. As expected, the mixing layer growth rate decreases with increasing velocity ratio. An approximately linear growth is indicated for $X \geq 100$ cm for all velocity ratios, although the correlation coefficient for the linear least squares fit (R) for the $r=0.9$ case is significantly lower (Table 2). Once again, this is not too surprising, considering the persistence of the wake in this case. It is interesting to note that the higher velocity ratio cases ($r \geq 0.7$), seem to exhibit a nominally linear growth almost from the start of their development.

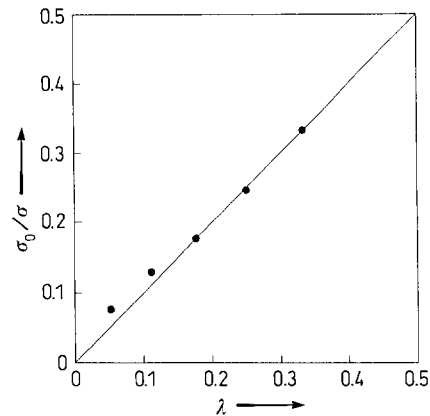
The virtual origin of the mixing layer (X_0) is well downstream of the splitter plate trailing edge at $r=0.5$, but then moves upstream with increasing velocity ratio, such that it is well upstream of the trailing edge at $r=0.9$, as illustrated in Table 2.

The effect of velocity ratio on growth rate has been determined analytically by Abramovich (1963) and Sabin (1965) giving the relation:

$$\sigma_0/\sigma = \lambda = (1-r)/(1+r) \quad (2)$$

Birch and Eggers (1973) and Rodi (1975) later correlated available data and confirmed the validity of this relation for fully developed subsonic mixing layers. They also showed that most of the other proposed empirical relations, such as those due to Miles and Shih (1968) and Yule (1971b) for example, tended to overpredict the growth rate at a given velocity ratio. As Rodi (1975) rightly pointed out, this discrepancy was almost certainly due to the higher free-stream turbulence levels in a lot of the earlier facilities (Rodi quotes Yule's free-stream turbulence level as 1% and that of Miles and Shih as 3%). High free-stream turbulence levels would lead to increased entrainment, and hence, a higher mixing layer growth rate, as convincingly demonstrated by Pui and Gartshore (1979), amongst others.

The measured growth rate of the mixing layer in the linear region for the present data is plotted as a function of velocity ratio, λ in Fig. 5. As suggested by Rodi (1975), rather than using the popular value of 11 for σ_0 , the appropriate value should be evaluated for each set of experiments. In the present case, it was found that a value for σ_0 of 10.5 best collapsed the growth rates for $r=0.5, 0.6$ and 0.7 on the theoretical correlation given by Eq. 2. This is consistent with Birch and Eggers' (1973) suggestion that the optimum value of σ_0 may be less than 11 for mixing layers with tripped initial boundary layers. The $r=0.8$ and $r=0.9$ cases have

**Fig. 5.** Effect of velocity ratio on mixing layer growth rate

slightly higher growth rates than predicted – this is attributed to the persisting wake effects rather than to free-stream turbulence, which was relatively low in the present case ($u'/U_e \sim 0.15\%$).

Therefore, as far as the mean flow properties are concerned, the mixing layers with $r=0.5$ to 0.7 seem to follow the established criteria for growth rate quite well. However, note that while a satisfactory mean flow behavior, in particular a linear growth of the mixing layer, is a necessary condition for the achievement of self-similarity, it is by no means a sufficient one – the behavior of the turbulence quantities is critical in determining self-similarity.

The profiles for the three measured Reynolds stresses (u'^2 , v'^2 and $u'v'$) are presented in Figs. 6–8. As far as the effects of velocity ratio are concerned, the qualitative trends for all three stresses are similar. For $r=0.5$, the peak Reynolds stress levels increase at first and good collapse of the profiles is observed beyond $X=111$ cm. Note also how all the profiles are symmetric about the mixing layer centerline for this velocity ratio. The profiles for $r=0.6$ and 0.7 also collapse well beyond $X=111$ cm, or perhaps even earlier, but initially the peak levels are higher than the asymptotic levels. The initial profiles are also asymmetrically distributed with a bias towards the low-speed side, although the symmetry is recovered by the third station at $X=73$ cm. In addition to the bias, the u'^2 and $u'v'$ data exhibit a smaller secondary peak on the low-speed side with the one in the primary shear stress data carrying a negative sign. Once a collapse of the profiles is achieved for a given Reynolds stress, the distributions for $r=0.5, 0.6$ and 0.7 appear to have the same shape and peak levels, thus suggesting that a common self-similar state has been attained for these velocity ratios. The initial Reynolds stress peak levels are even higher for $r=0.8$ and 0.9 ; the peak Reynolds stress level at the first streamwise location increases monotonically with velocity ratio. Although a gradual reduction is again observed with increasing downstream distance, a reasonable collapse of the profiles is not achieved within the measurement domain, thus clearly indicating a lack of self-similarity. A more sensitive indicator for self-similarity is the behavior of the

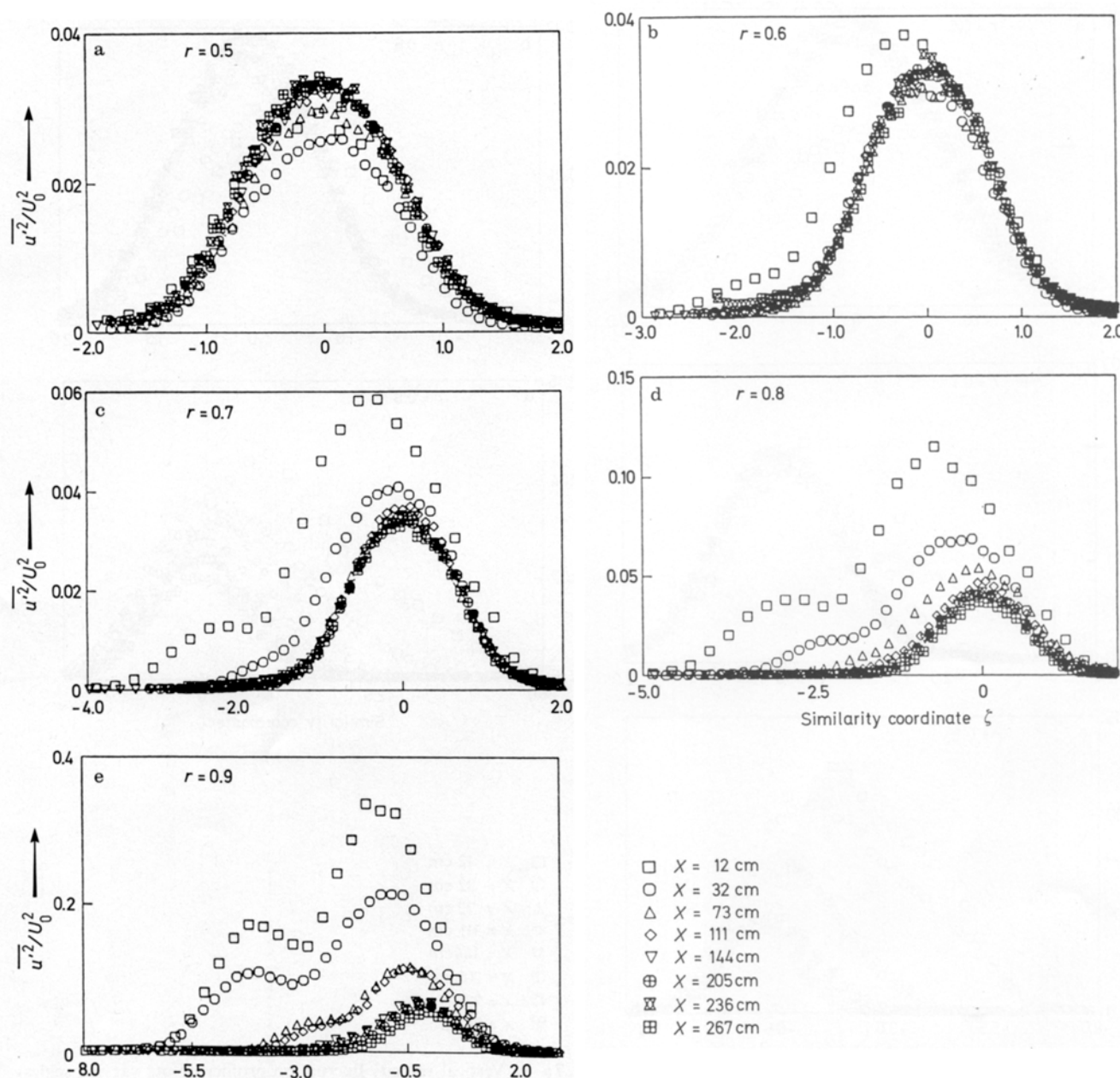


Fig. 6a–e. Streamwise velocity fluctuation profiles. (Note varying scales)

higher-order turbulence products which, in general, take even longer to exhibit self-similarity. These results (not presented here) further confirmed that for $r=0.5$ to 0.7 and $X \geq 111$ cm, the mixing layer had achieved a self-similar state.

The peak (measured) Reynolds stress levels are plotted as a function of streamwise distance in Figs. 9–11. As seen in the profile results above, the peak Reynolds stress levels for $r=0.5$ approach the asymptotic levels from lower values while those for the higher velocity ratios ($r \geq 0.7$) are found to approach them from higher values. The higher initial Reynolds stress levels are generally observed in the transition region of a mixing layer originating from laminar

boundary layers, but that is obviously not the case here since both boundary layers were successfully tripped. The peak levels for all the velocity ratios seem to achieve a more or less constant value at some point in the development. In fact, the $r=0.9$ case, and to some extent also the $r=0.8$ case, exhibit two distinct regions where the peak Reynolds stresses seem to achieve constant levels. After the initial rapid drop, the peak levels remain constant in the region between $X = 75$ to 125 cm, before dropping again to another lower level beyond $X \sim 150$ cm. Included for reference in these figures are peak Reynolds stress levels measured in a plane wake generated in the same facility under identical conditions (Weygandt and Mehta 1989). For the wake case, the boundary layers on the

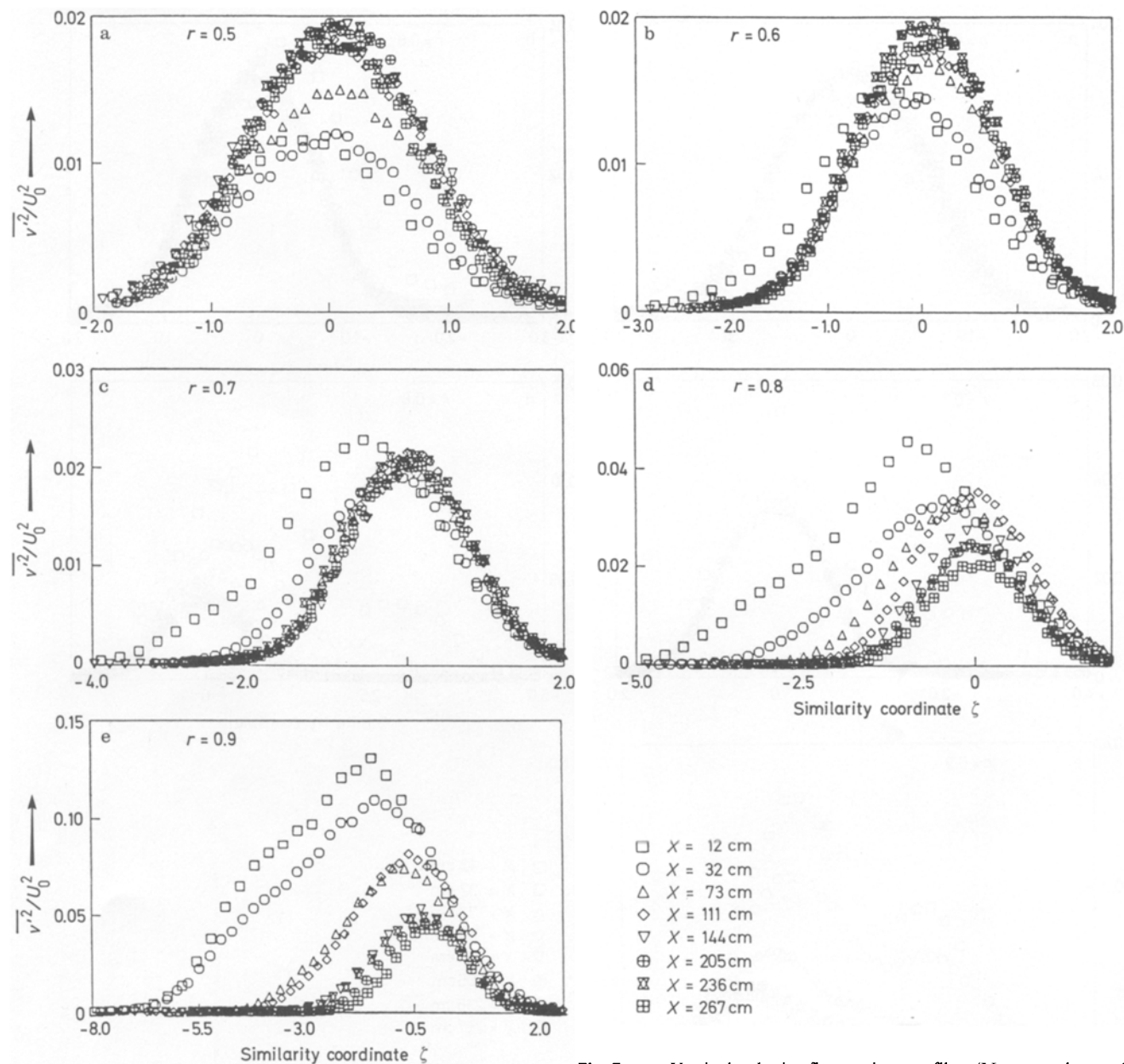


Fig. 7a–e. Vertical velocity fluctuation profiles. (Note varying scales)

splitter plate were also tripped and the free-stream velocity in the test section was 15 m/s; the wake data presented in Figs. 9–11 are normalized by the local velocity defect in the wake. The results from the wake study showed that the plane wake achieved self-similarity beyond $X \sim 100$ cm, as strongly suggested by the peak Reynolds stress distributions presented here.

For the $r=0.5$ case, the peak stresses appear to achieve constant levels beyond $X \sim 125$ cm. However, the peak values for the $r=0.6$ and 0.7 cases seem to reach constant levels a lot sooner; by $X \sim 100$ cm for $r=0.6$ and by $X \sim 75$ cm for $r=0.7$. The streamwise fluctuation ($\overline{u'^2}$) results show that the peak levels for these three velocity ratios asymptote to the

same value (within about 3%) of about 0.034 beyond $X = 150$ cm. The results for the vertical fluctuation ($\overline{v'^2}$) are similar, with the collapse to within about 9% for $x \geq 150$ cm. In this downstream region, the primary shear stress ($\overline{u'v'}$) appears to be a (weak) function of the velocity ratio, with the level increasing with velocity ratio. However, this variation of about 12% is comparable to the estimated measurement uncertainty for this quantity.

The relationship between the maximum shear stress and the self-similar growth rate implied by the conservation of momentum is given by Townsend (1976) for a two-dimensional mixing layer as:

$$-\overline{u'v'_{\max}}/U_0^2 = 0.141 \, d\delta/dX [(U_1 + U_2)/(U_1 - U_2)] \quad (3)$$

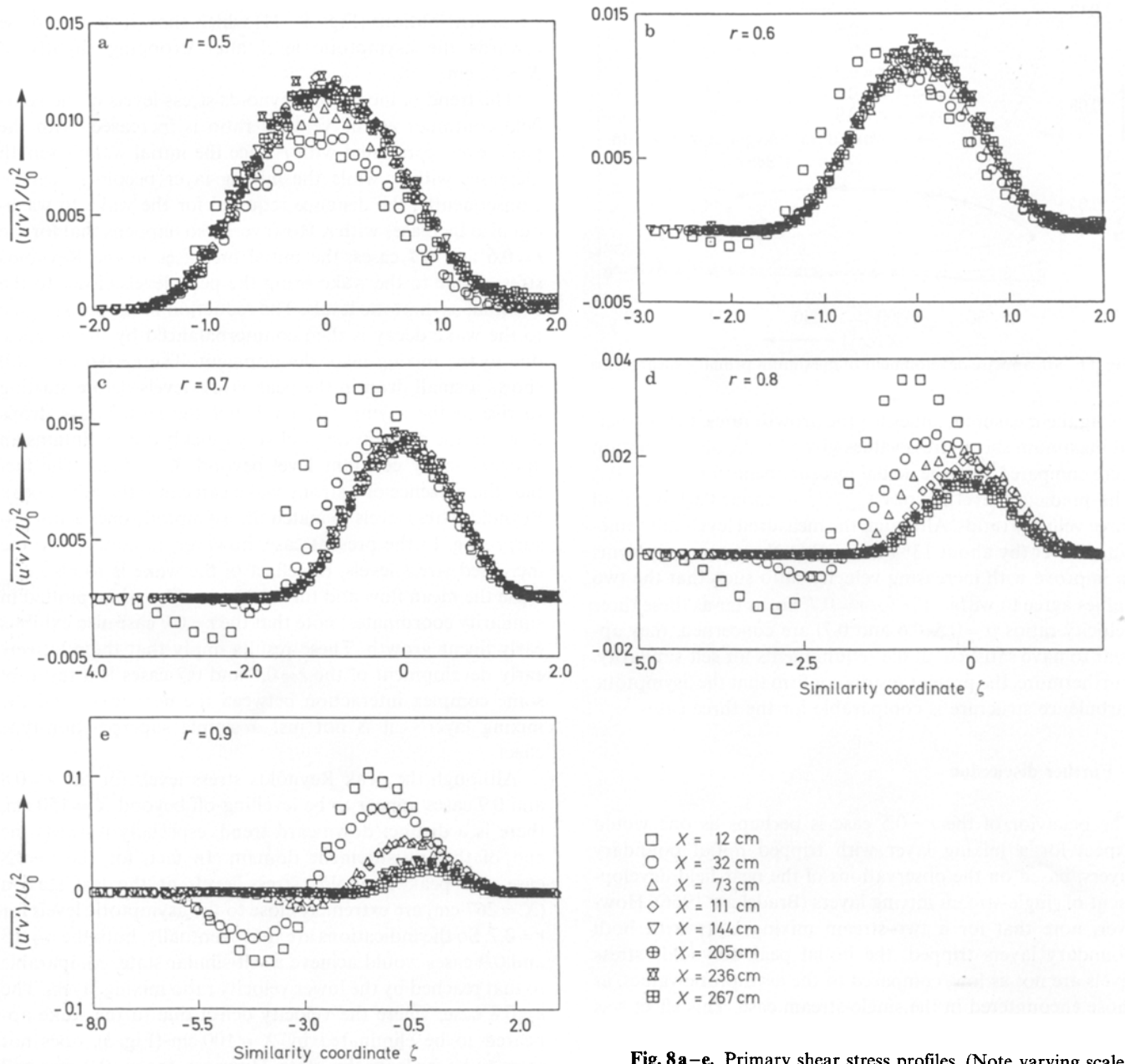


Fig. 8a-e. Primary shear stress profiles. (Note varying scales)

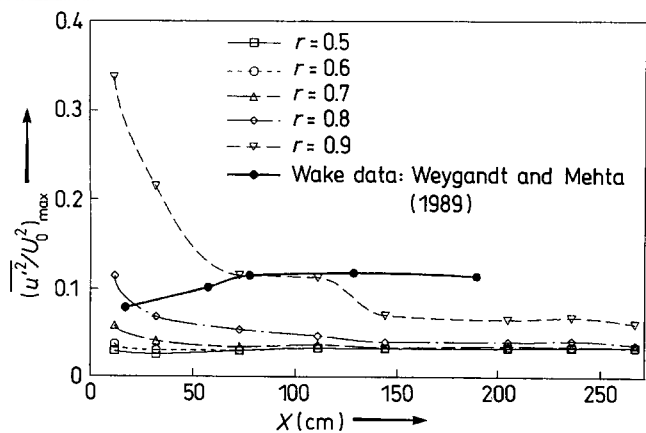


Fig. 9. Streamwise development of maximum streamwise velocity fluctuation

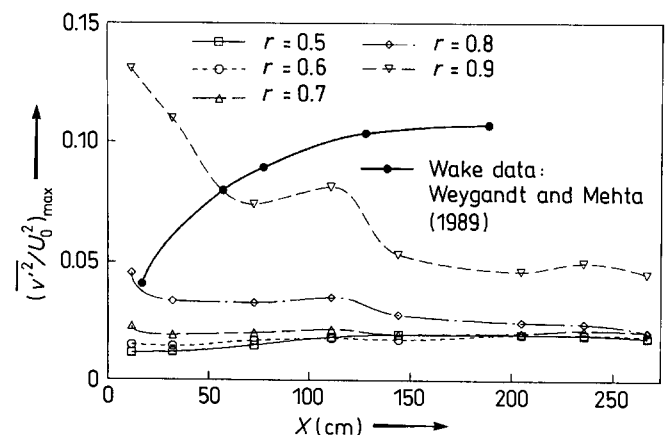


Fig. 10. Streamwise development of maximum vertical velocity fluctuation

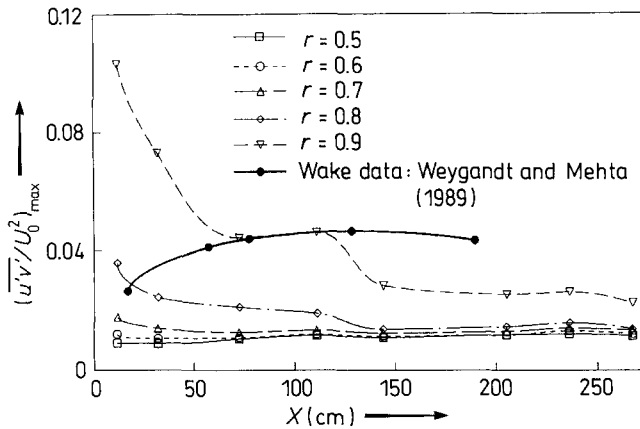


Fig. 11. Streamwise development of maximum primary shear stress

Using the measured values for the growth rates, the predicted maximum shear stress values given by the above relation were compared with the actual measurements for $r=0.5-0.7$. The predicted level for $(u'v')_{\max}/U_0^2$ is about 0.0134 for all three velocity ratios. Although the measured levels are somewhat lower (by about 13% for $r=0.5$), the agreement seems to improve with increasing velocity ratio such that the two values agree to within 1% for $r=0.7$. So, as far as these three velocity ratios ($r=0.5$, 0.6 and 0.7) are concerned, they appear to have satisfied all the requirements for self-similarity. Furthermore, the present results confirm that the asymptotic turbulence structure is comparable for the three cases.

4 Further discussion

The behavior of the $r=0.5$ case is perhaps as one would expect for a mixing layer with tripped initial boundary layers, based on the observations of the near-field development of single-stream mixing layers (Bradshaw 1966). However, note that for a two-stream mixing layer with both boundary layers tripped, the initial peak Reynolds stress levels are not as low, compared to the asymptotic values, as those encountered in the single-stream case. This effect was also observed in some of our earlier work on a tripped two-stream layer with $r=0.46$ (Mehta and Westphal 1986). The increased initial Reynolds stress levels are caused by the presence of the splitter plate wake. Apart from increasing the effective width of the overall shear layer, the addition of the wake also increases the mean velocity gradient across the mixing layer. The production of Reynolds stresses is, therefore, affected directly, noting that the primary production term for $u'v'$, for example, is $\bar{v}^2 \partial U / \partial Y$. The velocity defect due to the wake results in the secondary peaks (Figs. 6–8), whereas the increased gradient within the mixing layer leads to greater production, and hence higher peaks, towards the middle of the layer. This effect will continue until the wake is washed out by the mixing layer through entrainment, which for the $r=0.5$ case seems to occur very rapidly (by the second station at $X=32$ cm). This is apparent both, in the mean velocity results (Fig. 2), and in the peak Reynolds

stress distributions (Figs. 9–11) which show a slow increase towards the asymptotic level after dropping slightly at $X=32$ cm.

The trend of increased Reynolds stress levels in the near-field continues as the velocity ratio is increased, with the peak levels correlated with r since the initial wake strength increases with r , while the mixing layer becomes weaker. Consequently, the distance required for the wake to wash-out also increases with r . However, it so happens that for the $r=0.6$ and 0.7 cases, the initial increases in the Reynolds stresses due to the wake bring the peak levels closer to the eventual asymptotic levels. The reduction in the stresses due to the wake decay is then counterbalanced by the increase due to the mixing layer development. The $r=0.6$ case still shows a small drop in the peak stress levels before starting to rise to the asymptotic level, but the $r=0.7$ case drops down to the asymptotic level very quickly and maintains an approximately constant level beyond $X=75$ cm. The fact that the presence of a strong wake can cause the initial peak Reynolds stress levels to match the asymptotic ones is not too surprising. In the present case, however, in addition to the increased stress levels, the effect of the wake is to also collapse the mean flow and turbulence profiles when plotted in similarity coordinates; note that the $r=0.7$ case also exhibits early linear growth. These results imply that the relatively early development of the $r=0.6$ and 0.7 cases is a result of some complex interaction between the near-wake and the mixing layer – it is not just a simple superposition-type effect.

Although the peak Reynolds stress levels for the $r=0.8$ and 0.9 cases appear to be levelling-off beyond $X=150$ cm, there is a distinct downward trend, especially towards the end of the measurement domain. In fact, for the $r=0.8$ case, the peak Reynolds stress levels at the last station ($X=267$ cm) are extremely close to the asymptotic levels for $r=0.7$. So the indications are that eventually, both the $r=0.8$ and 0.9 cases would achieve a self-similar state, comparable to that reached by the lower velocity ratio mixing layers. The $r=0.8$ case, where the velocity defect due to the wake appeared to be eliminated by $X=100$ cm (Fig. 3), does not seem to be far from this state. However, the $r=0.9$ case still has remnants of the wake (Fig. 3) and would therefore be expected to take somewhat longer to achieve self-similarity. The first region of levelling-off, readily apparent in the $r=0.9$ data between $X=75$ to 125 cm, turns out to be rather interesting. The starting point of this region coincides with the point where the maximum peak stress levels in the mixing layer and those in the isolated wake are about equal. In fact, the $\bar{u}^2$ and $u'v'$ distributions are even coincident in this region. Furthermore, a closer look at Fig. 3 shows that the velocity defect decay for $r=0.9$ also has a discontinuity in this region, with the decay slowed down, as would be expected for a wake-type behavior. It therefore seems that, after the initial readjustment from boundary layer flow, the present free-shear flow is tempted to behave like a wake (temporarily), before the continuous shear maintained by the velocity

difference takes over. These observations make it clear that the wake can have a complex and lasting effect on the development of mixing layers, especially at the higher velocity ratios.

The present results are not really in contradiction with all the previous observations where the “self-similar” turbulence structure was found to be a function of velocity ratio. As discussed above in Section 1, the increase in peak Reynolds stresses with r reported by Yule (1971 a and b) and Saiy and Peerless (1978) was due to the high free-stream turbulence levels in their facilities. As pointed out by Abramovich (1963), the likelihood of free-stream disturbances affecting the mixing layer increases as the velocity ratio of the layer is increased. This is due to the fact that the local peak turbulence intensity in the mixing layer decreases with increasing velocity ratio (Saiy and Peerless 1978), as it must if the level normalized by the velocity difference is to remain constant. Hence, at the higher velocity ratios ($r \geq 0.5$), if the wind tunnel free-stream turbulence level is comparable to the maximum local level within the mixing layer, the growth of the layer will be dictated by the free-stream level which, in general, will not vary with velocity ratio. This effectively leads to a smaller change in the mixing layer characteristics with increasing velocity ratio and gives the effect of increasing peak Reynolds stresses when normalized by the velocity difference. In wind tunnels with low free-stream turbulence ($u'/U_e \leq 0.2\%$), such as the present one and the one used by Oster and Wygnanski (1982), this effect is minimized and the idea of a universal self-similar structure which is independent of velocity ratio becomes more realistic. The asymptotic $\overline{u'^2}/U_0^2$ level for $r=0.5$ to 0.7 in the present results (0.034) compares well (to within 5%) with the value reported by Oster and Wygnanski (0.032) for the range, $r=0.3$ to 0.6 . The averaged asymptotic levels ($r=0.5$ to 0.7) for all three measured Reynolds stresses in the present study also agree reasonably well with previous data obtained in the same velocity ratio range – to within 10% with the averaged measurements due to Spencer (1970), Pui and Gartshore (1979), Oster and Wygnanski (1982) and Mehta and Westphal (1986).

In an attempt to correlate the effects of velocity ratio and wake strength on mixing layer development, suitable normalizing parameters were sought for the velocity and length scales. For the velocity scale, an attempt was made to normalize the peak Reynolds stresses by the velocity difference plus the velocity defect. While this eliminated the drop-off in $u'v'_{\max}$ between the first and second stations for $r=0.5$ to 0.7 , the $r=0.8$ and 0.9 distributions still exhibited a non-monotonic behavior, with a peak around $X \sim 100$ cm. As for the appropriate length scale, Ho and Huerre (1984) suggested the initial instability wavelength (λ_n) which is proportional to the initial momentum thickness. Ho and Huang (1982) concluded that the high-speed side boundary layer momentum thickness (Θ_1) was the appropriate initial length scale since it controlled the most amplified frequency in the mixing layer. Browand and Troutt (1985) reached a similar

conclusion when they found that the growth rates for different velocity ratios collapsed when normalized by Θ_1 . However, since in the present case, the high-speed side velocity, and hence boundary layer properties, were kept constant for all velocity ratios, Θ_1 would obviously not be adequate in collapsing the development distance. Another parameter which may be considered is the virtual origin of the mixing layer (Table 1), which does move upstream with increasing velocity ratio. However, it was found that accounting for the virtual origin in Figs. 9–11 was still not enough to shift the data adequately, and the dependence of development distance on velocity ratio was maintained. Hence, the additional effects due to the splitter plate wake seem to further complicate the already complex behavior of this fundamental free-shear flow. This is especially borne out by the non-monotonic behavior of the mixing layer development distance with velocity ratio.

5 Conclusions

An experimental study has been conducted on the effects of velocity ratio on plane mixing layer development. A velocity ratio range of between 0.5 and 0.9 was investigated with both initial boundary layers tripped. The results indicate that the splitter plate wake plays a very dominant, and in some cases a lasting, role in the development of the mixing layer. For the higher velocity ratios ($r=0.8$ and 0.9), the mixing layer did not achieve self-similarity within the measurement domain, although a slow approach to it was apparent. A self-similar state was achieved by the mixing layers with velocity ratios of between 0.5 and 0.7 satisfying all three of the necessary conditions: namely, linear growth, asymptoting of the peak Reynolds stresses to constant values and collapse of the mean flow and turbulence profiles when plotted in similarity coordinates. Furthermore, the asymptotic turbulence structure for these three velocity ratios was found to be very similar. The present study confirms previous suggestions that the measured turbulence structure in a self-similar mixing layer will be independent of velocity ratio only if the free-stream turbulence levels in the wind tunnel are relatively low ($u'/U_e \leq 0.2\%$). The data suggest that the development distance to self-similarity decreases with increasing velocity ratio, between $r=0.5$ and 0.7 , after which the trend is reversed so that it increases with r . This effect is mainly caused by the wake increasing the turbulence levels in the near-field of the mixing layer, with the increase rising with r . In the case of the $r=0.7$ mixing layer, these increased levels coincide with the asymptotic levels for self-similarity. However, it should be noted that the overall effect of the wake on mixing layer development is rather complex and cannot be accounted for through any simple scaling.

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