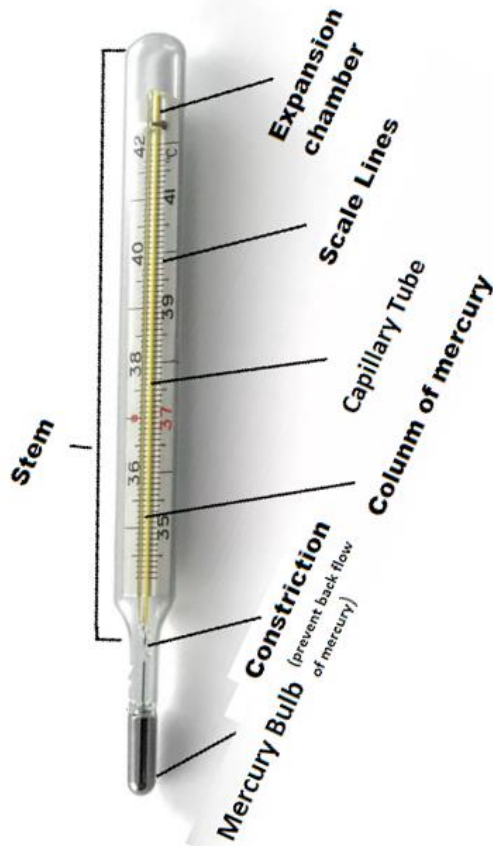


Solution to Problem 3 of Problem set -1

Problem Statement & sample Solution: Part -1

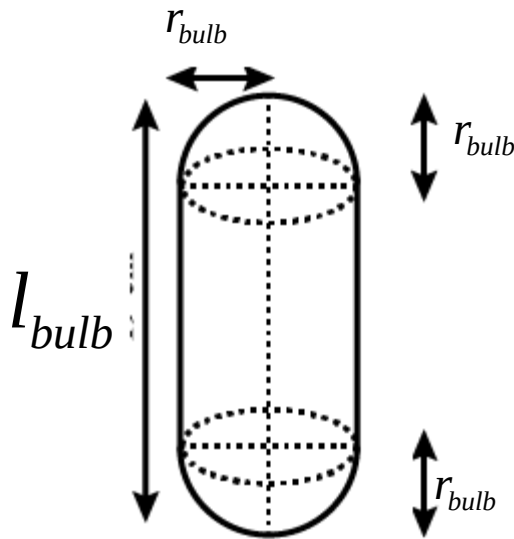
- Prove that ***Zeroth law*** of thermodynamics leads to design and selection of a thermometer with an appropriate value of time constant for each application.



- Study the picture given below and identify/recognize following design features:
- The glass thickness in the stem portion is very high.
- Glass thickness is very low in the bulb portion: This facilitates the bulb to reach thermal equilibrium as fast as possible with the system.
- The shape of the bulb is optimized to a maximum feasible surface area.

Technical Information

- An appropriate value of time constant for each application.

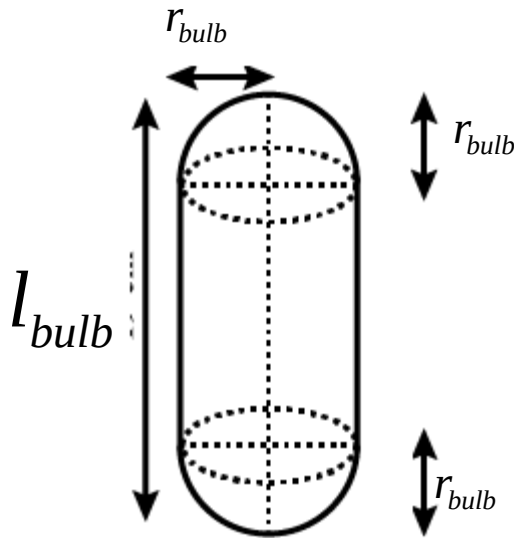


- The bulb of a thermometer may be approximated as cylinder with hemispherical ends.

$$V_{bulb} = \pi \left\{ r_{bulb}^2 l_{bulb} + \frac{4}{3} r_{bulb}^3 \right\}$$

Problem Statement & sample Solution: Part -1

- Prove that ***Zeroth law*** of thermodynamics leads to design and selection of a thermometer with an appropriate value of time constant for each application.



- At room temperature, maximum amount of thermometric fluid (*tf*) mercury will be inside the bulb.

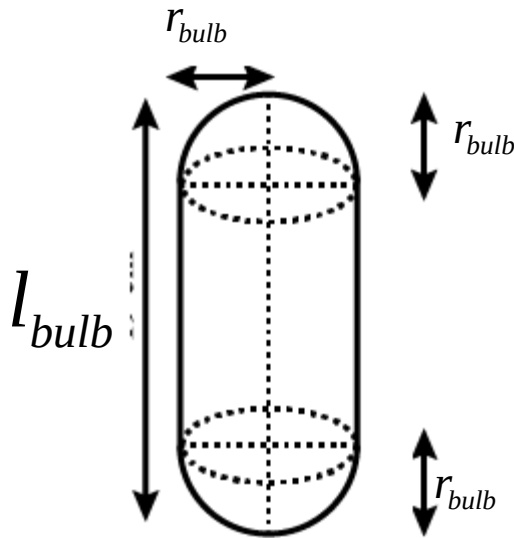
$$\therefore V_{tf-T_{room}} \approx V_{bulb} = \pi \left\{ r_{bulb}^2 (l_{bulb} - d_{bulb}) + \frac{4}{3} r_{bulb}^3 \right\}$$

- Volume of *tf* at any unknown temperature, T is:

$$V_{tf-T} = V_{bulb} \{1 + \gamma \times (T - T_{room})\}$$

Technical Information

- Value of time constant for each application.



- The time constant of thermometer is better approximated as:

$$\tau = \frac{\rho_{mercury} V_{bulb} C_{mercury}}{UA_{bulb}}$$

System heat transfer coefficients (U) in W/m²K @ 20 °C

Medium	Speed, m/s	Diameter, mm							
		0.25	0.5	1.0	1.5	2.0	3.0	4.5	6.0
Water	0.4	29 500	20 700	14 600	11 900	10 300	8 400	6 860	5 940
	0.2	21 100	14 700	10 300	8 430	7 290	5 940	4 850	4 200
Air	2	327	224	155	125	108	87	71	61
	1	242	163	112	90	77	63	51	44

properties of thermometric fluids

Popular thermometric fluids: mercury & alcohol

Material	Coefficient of Volumetric Thermal Expansion γ	Density at 0°C	Specific heat
Mercury	180×10^{-6} per°C	13594 kg/m ³	0.14 kJ/kg°C
Alcohol	1120×10^{-6} per°C	789 kg/m ³	2.4 kJ/kg°C

Problem Statement & sample Solution: Part -2

- Two thermometers using mercury and alcohol having same diameter of capillary tube (0.5mm) and same amount of increase liquid column (100mm increase in liquid column for an increase in temperature of 40°C) are considered for study.

$$\Delta V_{tf} = \gamma \times [\Delta T] = \gamma \times [40]$$

$$\Delta V_{mercury} = V_{bulb-mercury} \times 180 \times 10^{-6} \times [40]$$

$$\Delta V_{alcohol} = V_{bulb-alcohol} \times 1120 \times 10^{-6} \times [40]$$

Problem Statement & sample Solution: Part -2

- Two thermometers using mercury and alcohol having same diameter of capillary tube (0.5mm) and same amount of increase liquid column (100mm increase in liquid column for an increase in temperature of 40°C) are considered for study. The bulb length is equal to four times the bulb diameter.

$$\Delta V_{tf} = V_{bulb} \times \gamma \times [\Delta T] = V_{bulb} \times \gamma \times [40]$$

$$\Delta V_{mercury} = \pi \left\{ r_{bulb}^2 (l_{bulb} - d_{bulb}) + \frac{4}{3} r_{bulb}^3 \right\}_{mercury} \times 180 \times 10^{-6} \times [40]$$

$$\Delta V_{alcohol} = \left\{ r_{bulb}^2 (l_{bulb} - d_{bulb}) + \frac{4}{3} r_{bulb}^3 \right\}_{alcohol} \times 1120 \times 10^{-6} \times [40]$$

$$\text{Given } \Delta V_{mercury} = \Delta V_{alcohol}$$

Problem Statement & sample Solution: Part -2

- Two thermometers using mercury and alcohol having same diameter of capillary tube (0.5mm) and same amount of increase liquid column (100mm increase in liquid column for an increase in temperature of 40°C) are considered for study. The bulb length is equal to four times the bulb diameter.

$$\text{Given } \Delta V_{\text{mercury}} = \Delta V_{\text{alcohol}}$$

$$\frac{\Delta V_{\text{mercury}}}{\Delta V_{\text{alcohol}}} = \frac{\left\{ r_{\text{bulb}}^2 (l_{\text{bulb}} - d_{\text{bulb}}) + \frac{4}{3} r_{\text{bulb}}^3 \right\}_{\text{mercury}}}{\left\{ r_{\text{bulb}}^2 (l_{\text{bulb}} - d_{\text{bulb}}) + \frac{4}{3} r_{\text{bulb}}^3 \right\}_{\text{alcohol}}} \times \frac{18}{112} = 1$$

$$\frac{\left\{ r_{\text{bulb}}^2 (l_{\text{bulb}} - d_{\text{bulb}}) + \frac{4}{3} r_{\text{bulb}}^3 \right\}_{\text{mercury}}}{\left\{ r_{\text{bulb}}^2 (l_{\text{bulb}} - d_{\text{bulb}}) + \frac{4}{3} r_{\text{bulb}}^3 \right\}_{\text{alcohol}}} = \frac{112}{18}$$

Problem Statement & sample Solution: Final part

- These two thermometers are suddenly immersed in a system with value of heat transfer coefficient, **$U = 400 \text{ W/m}^2\text{°C}$** (proportionality constant in Newtons law of cooling).
- Carryout appropriate calculations and select a thermometer which demands less time to measure temperature of a system.

$$r_{\text{bulb-mercury}} = 1.84 r_{\text{bulb-alcohol}}$$

$$\tau_{\text{mercury}} = \frac{\rho_{\text{mercury}} V_{\text{bulb-mercury}} C_{\text{mercury}}}{UA_{\text{bulb-mercury}}}$$

$$\frac{\tau_{\text{mercury}}}{\tau_{\text{alcohol}}} = \frac{\rho_{\text{mercury}} V_{\text{bulb-mercury}} C_{\text{mercury}}}{\rho_{\text{alcohol}} V_{\text{bulb-alcohol}} C_{\text{alcohol}}} \times \frac{A_{\text{bulb-alcohol}}}{A_{\text{bulb-mercury}}}$$

Problem Statement & sample Solution: Final Part

Material	Coefficient of Volumetric Thermal Expansion γ	Density at 0°C	Specific heat
Mercury	$180 \times 10^{-6} \text{ per } ^\circ\text{C}$	13594 kg/m^3	$0.14 \text{ kJ/kg}^\circ\text{C}$
Alcohol	$1120 \times 10^{-6} \text{ per } ^\circ\text{C}$	789 kg/m^3	$2.4 \text{ kJ/kg}^\circ\text{C}$

$$\frac{\tau_{\text{mercury}}}{\tau_{\text{alcohol}}} = \frac{13594 \times r_{\text{bulb-mercury}} \times 0.14}{789 \times r_{\text{bulb-alcohol}} \times 2.4}$$

$$\frac{\tau_{\text{mercury}}}{\tau_{\text{alcohol}}} = \frac{13594 \times 1.84 \times 0.14}{789 \times 2.4} = 1.85$$

Alcohol thermometer which demands less time to reach thermal equilibrium. This must be selected.....