Tribology

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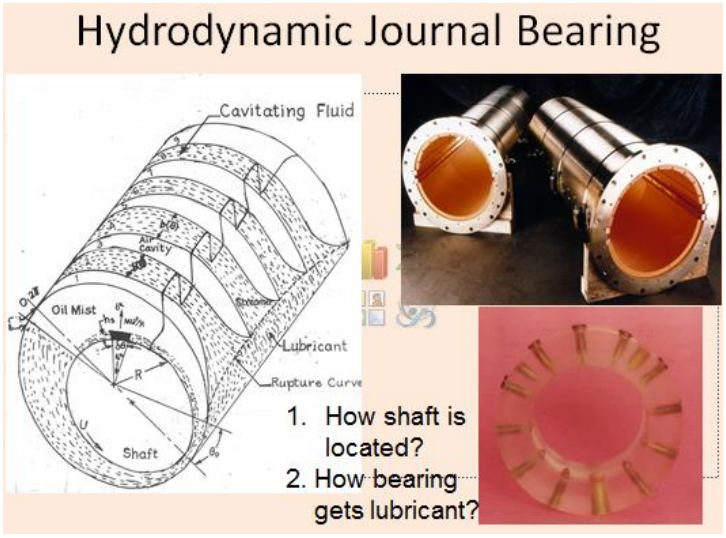
Module No. # 06
Lecture No. # 41
Hydrodynamic Journal Bearings

Hello and welcome to 41 st lecture of video course on Tribology, topic of the present lecture is Hydrodynamic Journal Bearings. We have studied, what is the meaning of hydrodynamic action? It is basically governed with the velocity, relative velocity and tries to pump, the lubricant using the velocity is more like a pumping action, because of the rotation. Sometimes, we call this section as self acting, because there is certain rotation, we are getting lubrication mechanism except from cost point of view, this mechanism turn out to be cheapest, if there is a lubricant supply; however, many times we couple hydrodynamic action with hydrostatic action.

In previous lecture, we discussed about hydrostatic lubrication, hydrostatic lubrication has all advantages except the cost, as we require pump, we require tubing, we require extra manufacturing steps or we say that, very precision manufacturing steps. The cost is a major drawback of hydrostatic lubrication and the cost is major advantage of hydrodynamic lubrication.

So, often these two mechanisms are coupled, hybridized and most often, hydrodynamic bearing which has been used to start with hydrostatic action and subsequently done to the hydrodynamic action. That gives overall benefit from the cost, running cost particularly and low coefficient of friction almost zero wear, no wear, because when we are coupling hydrostatic with hydrodynamic, there will not be any wear rate; however, if we remove hydrostatic believe only on hydrodynamic lubrication, there is a possibility of wear at a start and stop, when the relative velocity is negligible, small, initially. So, we are going to discuss, we are going to start hydrodynamic lubrication mechanism using or we say that, applying on journal bearings.

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This is the typical naval bearing we say that, they are processed or these are the bearings hardened properly. So that they do not get wear out, they have the hardness and they can be operated with the water lubricant or we say, the water acts as the lubricant gives the film thickness, separates the shaft from bearings surface; however, we can say this bearing is cylindrical shape and particularly very long bearing.

Here, length to the diameter ratio is 4 very large length, there is a possibility of other lengths also, smaller lengths, often used in automobiles, because of the space constraint, space restriction and again this kind of bearings are used in smaller size with the length to diameter ratio even can be 0.1 and 0.2 and what we are showing here the ratio as a 4.

Exceptional cases, when their turbines blades or turbine shafts are very long and we required many bearings, many bearings in many housing and they want to get rid of those housing, lesser manufacturing steps or we say, lesser assembly steps in those situations long bearings are acceptable, keep overall benefit. However, long bearing are out dated concept we generally use a short bearing concepts.

Before we start hydrodynamic journal bearing, I want to see what happens exactly inside the bearing. When we think about what happens exactly inside the bearing, then image appears something like there will be lubricant, something like dotted lines over here, because of the shaft rotation and then, there will be diversion domain as available liquid, which is incompressible goes in larger area, there will be discontinuity in liquid lubricant.

Available liquid is lesser than available space, naturally air will occupy remaining space and because of the rotation, because of the viscous effect, often this liquid lubricant gets streamed or we say that, number of streams are combined together and show some cavity in between, you can think the whole liquid lubricant together at the centre and both the sides of bearing

occupied by the air, but it does not appear, when we watch operation of hydrodynamic bearing.

Question comes, how to watch this? This bearings are not transparent, this can be hypothesized we can think however, but how to prove it, then there is a provision we can make this kind of bearing with a transparent material and that is shown here, we have used thermocouple heating at the centre, but it looks like thermocouples are the settings, but these 40 mm thickness of the bearing and at the centre around 20 mm this is a 3 to 4 mm rivet, which is used as a thermocouple heating can be seen and when we support shaft, we can see this kind of streamers.

You can see the separation between the two lubricant columns and separation is with air cavity. We are also able to find out this kind of rupture curve, lubricant gets ruptured over here and we are able to see this diversion domain is occupied with here, it will be difficult to say it is absolutely only in the air, there is a possibility of some oil, lubricant particles getting dissolved or getting mixed with air and making some sort of oil mist that is why we are giving option, all options, all mist streamers, air cavity, rupture and to model, all these cavitations.

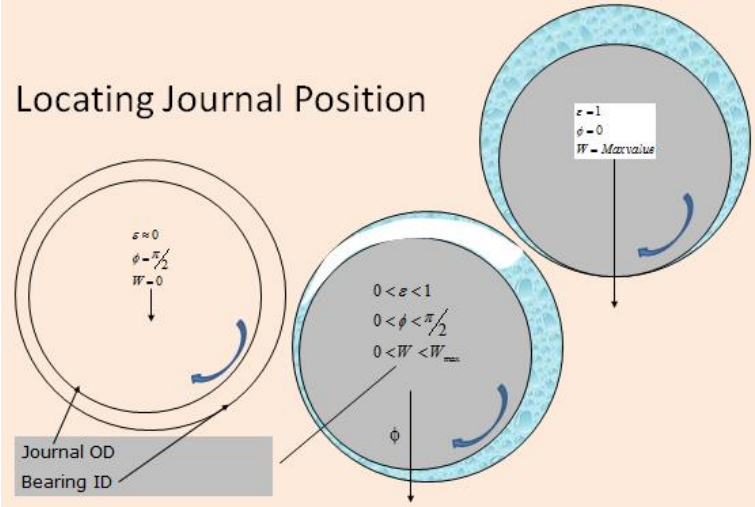
We require good mathematical treatment, more exhaustive calculations, which we are not going to discuss, but this kind of picture, this kind of sketch is able to show that, bearing is only half loaded, upper half, not exactly upper half, but almost 180 degree is unloaded or pressure generated in this diversion domain is negligible or may be in the vacuum below that mass with ratio.

And that leads to think only half of the bearing is useful, other half bearing is not that much useful, it gives continuity to flow. If we do not have upper half of this bearing, naturally this flow will not be circulated, we will be requiring too much lubricant, but here lubricant is going to be pumped. Almost complete lubricate is going to get waste, it is not getting recirculated; the lubricant flow rate will increase in this case.

So, just we are having this kind of thinking, naturally there will be some questions coming in our mind, first thing there is a clearance between a shaft and the bearing surface, this is a basically clearance and within this clearance, oil is getting recirculated or circulated. So, how the shaft decides this position! Why, shaft gets located at on only one position. Shaft can get disturbed here and there, make a different curve, but the way it had been shown, it shows that the shaft is getting located at one position. The question comes, how shaft decides its location that is the one and as for the question is imposed to you, how shaft is located second thing, how lubricant is getting pumped in this.

We are able to see, there is a liquid lubricant over here, lesser lubricant in this side, but from where the liquid lubricant is getting pumped, it is not been shown in this, what kind of arrangement will be used to pump the liquid lubricant. So, these two questions need to be gone through before thinking designing of journal bearing, hydrodynamic journal bearing.

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So, let us think about the first question, locating journal position, I will draw I am showing a sketch assuming the shaft is a steel surface and solid shaft, then showing only the bearing inner surface, some sort of the droplet is mixed with this, we have clearance the space is occupied with liquid lubricant.

One equation is being shown over here this says epsilon is equal to 1, this is the eccentricity divide by clearance, radial clearance and that is the maximum and we say that, shaft is at the maximum displacement and touching the surface of bearing, for operation we do not require this because, when the bearing and the shaft surface are in touch naturally, there will be solid to solid contact and that will lead to react.

Then, other one is the phi is equal to 0 question come, what is this phi of course, we say this is attitude angle, but there is no angle in this and last one say, W is maximum. This kind of situation occurs, when applied load is at extreme level, is a maximum volume again think of whatever the load you have apply and if there is no rotation, shaft is going to touch this surface of the bearing. So, this is again a subjective depends on operating condition, if there is more and more rotation, there is more and more possibility of W, if there is no rotation, even 1 gram, 2 gram, 1 milligram that will be sufficient for shaft to go and touch the bearing surface.

So, this is the relative term. It is the subjective term and depends on the viscosity depends, on the rotation of liquid lubricant and shaft rotation. Now as we are saying that, this is depending on the shaft rotation, lets rotate this shaft, if shaft starts rotating what will happen, this leads to rotate with the shaft surface, if that happens more and more liquid is getting pumped this side and released from this side and that will and shaft is in a position naturally, shaft will slightly levitated and the shaft centre will be directed towards this side and that is mechanism, how shaft is getting located. Say, whether there are rotation, naturally this position is passing the liquid, because we require some space our liquid will not be pumped, will not be transferred from one surface to other surface, from one side of the shaft to the other side.

Naturally, it has to give some clearance some space and that is the levitation by hydrodynamic action. Naturally this position how do we say, this line is showing a connection between bearing and journal centre and this is the load line, load is been applied in this direction, angle between line of centre and load line will be known as a attitude angle that is the reason, why we are showing attitude angle 0 in this position. Line of centre and the load line, they are in the same position, no angle, 0 angle while because of the rotation, shaft centre is getting shifted from its original position and getting levitated.

Naturally, eccentricity will come slightly lesser than 1, but much greater like 0.4, 0.3, 0.2, but there will be more than 0, if load is been applied. And this load, final load will be lesser than W max reason being, there will be some load generated by liquid, which is going to oppose this shaft and that is the equilibrium force, apply load minus force generated will be lesser than the maximum load at any of this position.

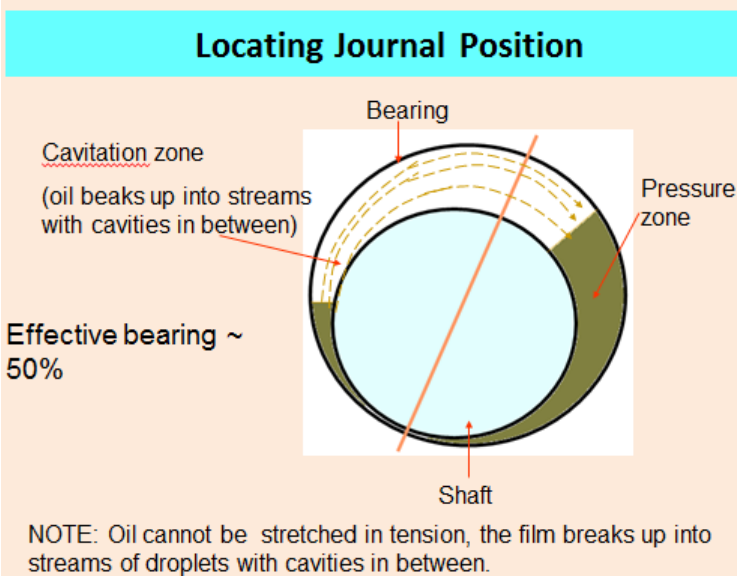
So, these are the two conditions, which is epsilon is one, shaft is completely centric touching the bearing surface, even it is not touching the bearing surface that is why eccentricity ratio is lesser than 1 and again is not coming to the bearing centre. So, it is greater than straighter, attitude angle here is phi is 0, but it is does not reach the maximum 90 degree. So, this angle will be between 0 and pi by 2 that is 90 degree.

Now, that situation is also passed, you see, because of the shaft rotation and I suddenly remove the load, there is no load on a shaft surface, what will happen, this shaft is going to rotate and gets location at bearing center as there is no load, this shaft and getting centre will come in this horizontal line that is what is being shown as pi by 2.

There is a possibility of eccentricity slightly greater than 0, but it can be approximated, its epsilon is approximately equal to 0.0 this is why, depends on the rotation, depends on the load and depends on the lubricant kind of liquid we are using.

Shaft gets located and overall design we need to think from the eccentricity point of view or eccentricity ratio point of view and attitude angle point of view, if attitude angle is very large, naturally shaft is going to fluctuate it will not remain in a steady position that is why we are saying that, the eccentricity also will continuously change. It is more like we have 20 dollars or 20 coins and one coin is coming plus minus so naturally 5 percent variation, but we have 100 coins and one coin plus minus so only 1 percent, if we have 1000 coins and one coin plus minus so 0.1 percent same situation here. When eccentricity is very high even in slight turbulence is not going to change much, but if eccentricity is very low, slight turbulence going to create in stability, continuous changing its position, shaft is going to change its position.

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In other words, there are different ways to represent that, when we talk about the journal bearing, there will be shaft surface, there will be bearing surface and there will be convergent domains like divergent domains this will be occupied by liquid lubricant.

In divergent domain, there will be number of streamers separated by cavities that is why we know, this kind of zone can be treated as a cavitation zone and this position start a pressure zone from the maximum film thickness position, this angle will be decided based on what kind of feeding arrangement or supplier lubricant is being arranged. If we are providing some

oil groove over here, some feeding over here, then naturally liquid lubricant will be pumped from here and this will be occupied by most of the liquid.

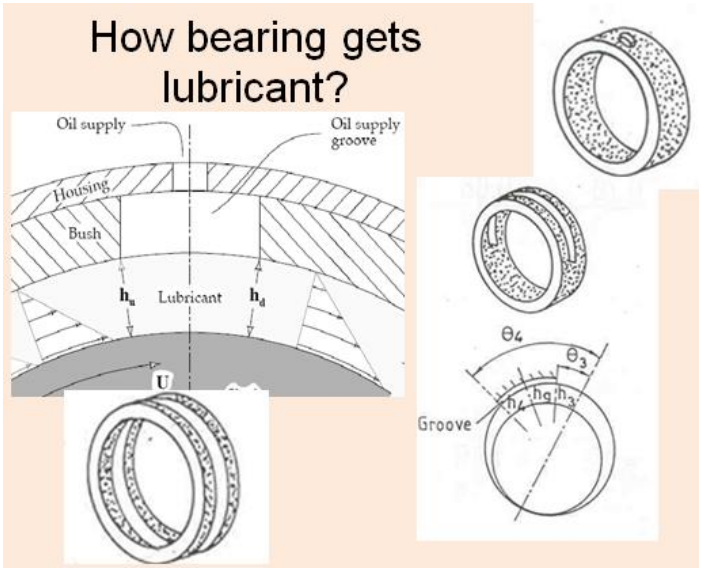
But if we are providing liquid lubricant, feed hole over here, naturally this position will remain cavitated depends on the location I will say the location of the feed hole, bearing operation will be slightly different, over all equilibrium will be different, temperature rise also will be different, it is important to find out what kind of feed hole arrangement we are providing and what is the location of that.

Why this streamers are being formed reason being, oil or liquid lubricant cannot be stretched in tension or attention they will stop it, then there is a lesser pressure vacuum naturally tension is going to develop in the liquid and that, that is why they say that, liquid lubricant will break down into streamers or streamers of droplets having cavity in between those streamers.

And second conclusion, which was also represented in the previous slide we say that, bearing is effective only to 50 percent, I cannot say exactly the 50 percent, there is slightly more, this space zone is more than 180 degree, but lesser than 210 degree.

So, not exactly 50 percent, but portion which is beyond 180 degree will not be able to show lot of contribution to other load sharing. So, for simplicity or easy calculation we can assume bearing is a factor only for 50 percent of its circumference, 50 percent of its circumference I am not talking about 50 percent of the length, half of the bearing is from circumference point of view.

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Now, how to feed this bearing and how to pump this lubricant we say bearings will be placed in housings and housing will have some oil supply hole. I can think about the through and through groove or oil hole a drill within bearing housing and bearing surface or I can think about supply at a smaller size hole, but we are keeping larger size pocket. So, the turbulence is negligible, liquid is supplied and we have abundant supply for a liquid. So, bearing does not get disturbed like a lubricant. So, this is possible and depends on the requirement, we can think of the smaller dimension of this cavity, larger dimension of complete cavities. So, there are three possibilities we say that, bearing also has a hole, housing also has a hole and this holes are going to be co linear or we say that, their axes are going to match, this is one possibility.

There is another possibility the way it is being shown that, groove is extended in this case is being showing as 180 degree, but there is a possibility may be slightly different and in this particular is shown that, groove is starting before maximum film thickness line or before if I am assuming the rotation in this direction is a clock wise direction, then oil hole arrangement or you say can that groove arrangement is before the start of this line of centre or maximum film thickness.

In that situation this position, this complete position will be occupied by liquid lubricant ever if you place this groove somewhere here, naturally not complete position will be occupied only some position of the bearing will be getting liquid lubricant and the groove extend can be over this theta can be theta 3 can be make negative side also, it can be 0 also and can be any value, while theta 4 will be slightly more than theta 3 that is why they are showing some extent out groove.

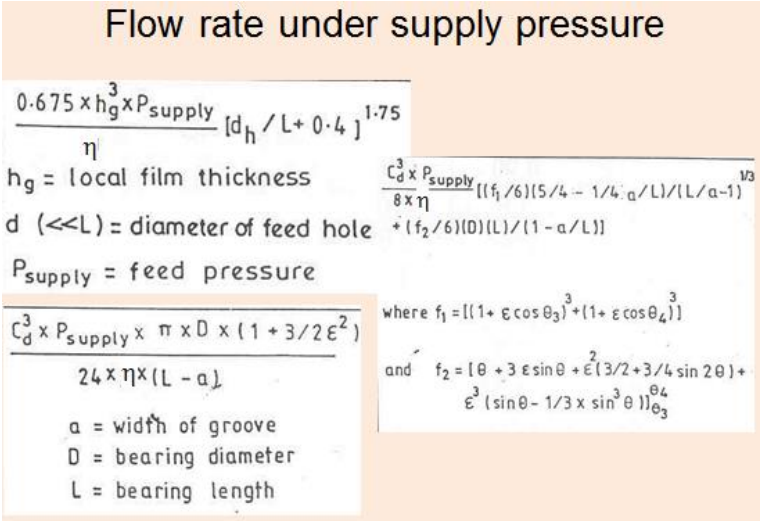
But there is another possibility say whole circumferential groove in this, first think in the mind, no this is not possible, because this will be acting as a two bearings. if I come completely naturally the bearing is not a single entity, it will be in two pieces, there is one possibility like that or other one is that, we do not cut from outer surface, but this at the some cavity will be here or there is a not through and through slot in this, upper surface is covered having may be the thickness of 3 mm and at the depth there is a groove.

If I assume a 10 mm thickness of the bearing, 3 mm groove cannot accept 1 or 2 slot the way to match this oil supply will have a drill over here and below 3 mm from the surface outer

surface, the groove will start a the groove depth will be 7 mm. If I am assuming the 10 mm of thickness of this bearing in that case, 3 mm will be thickness outside and 7 mm will be depth of pocket.

There is a possibility we can keep any arrangement, if we require very low flow rate, lesser supply of oil in those situations, we will prefer this oil arrangement, because the weakening effect is minimum, weakening in a strength of this bearing material will be minimum, while in this case weakening effect is slightly more say higher side, while in this case is a maximum size and if I clear and we say when we do analysis we frankly we assume the bearing length is lesser than half of the bearing and we consider these two bearing as this as two bearing instead of one bearing for bearing analysis.

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Now, there are some equation we can find out oil flow rate using finite difference method through schemes, but there are well established equation available for these three arrangements, single oil hole, groove extent being may be same 30 degree, 40 degree, 50 degree, 100 degree, 120 degree depends on the requirement and finally 360 degree. So, this equation is for oil hole.

And that is why we say that, wherever the groove is there, what is a film thickness here you can see it if the sensitivity is maximum it's in a cubic form larger the value of h g, larger will be flow rate anyway if we increase the supply, ratio of flow rate also will increase, but you can see the sensitivity flow rate of supply pressure is not that much dominating effect compared to this dominating effect. The viscosity will also play a role, larger the viscosity

lesser will be flow rate that is the definition of the viscosity, it gives the more and more resistance to other flow, larger the viscosity naturally will give more flow, lesser flow rate.

And this is the geometry parameter d_h is the diameter of the whole divide by length, assuming d_h will be much smaller in the length or you say compare to the length it may be the lesser than 10 percent even based on this we can find out, what will be flow rate. Where we will be using flow rate naturally will be using flow rate when we talk about temperature rise, a major purpose of high flow rate is to reduce the temperature rise.

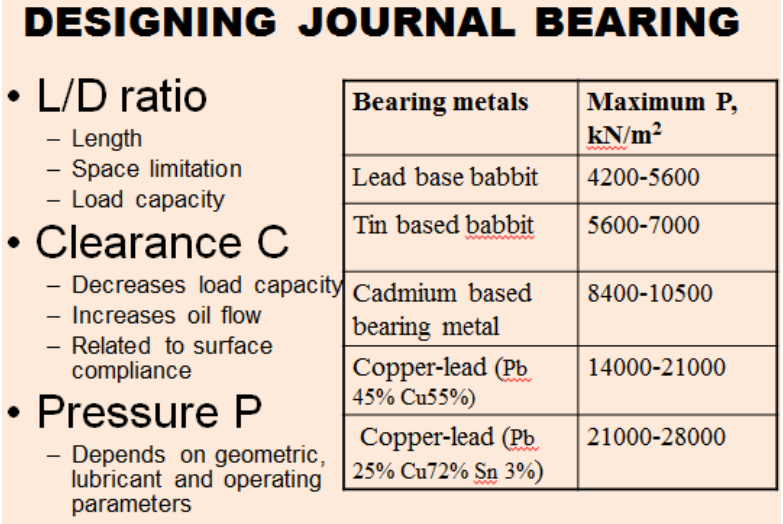
Now, for the complete groove, 360 degree groove we are able to get highly more tds terms in this case, we are writing L minus a; that means, we are talking about the width of the groove, the length is 50 mm, width is a 5 mm naturally effective length is a 45 mm, then we are talking all is C_d here instead of h_g, C_d plays an important role of course, when we talk about h_g it will come indirectly, h_g will be represented as C in that case 1 plus epsilon cos θ.

Similar way C_d is coming in this option in this relation and addition there is a viscosity effect also. So, supply pressure or we say that, if I know the supply pressure, flow rate can be calculated for 360 groove, circumferential groove at the mid plain by using this relation, more generic more general is third equation, which is for the partial groove, a theta 3 and theta 4 are going to decide, what will be the angular extent of this groove.

Again in this case the C_d is appearing and this is the maximum sensitivity, supply pressure, viscosity, f_1 and f_2 and f_1 and f_2 can be determined based on the film thickness or we say that, shaft position, finding eccentricity and find out what will be the separation from the mean line or from the line of centre, we are assuming that as going to impart and that going to be playing a very important role, line of centre. So, once we know θ 3, θ 4 we can find out, f_1 and f_2. Once we know f_1 and f_2 we can find out, what will be the flow rate for partial groove.

So, these relations are well established by the martin somewhere in 1983 and we used commonly in number of books and in our lecture also we can utilize his relations, if we find temperature rise is significant, temperature rise is well within the bound we may not require high pressure, supply pressure is just above the atmospheric ratio in those situations, flow rate due to this arrangement is not going to play major role, it will be the flow rate due to hydrodynamic action it has been the dragging action, shaft when a shaft rotates and there is a viscous liquid, shaft will pull that liquid along with the shaft surface.

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Now, you can think about the designing journal bearing, we have three parameters, what is the length that is the length to diameter ratio, what should be the clearance, what should be the radial clearance and how much pressure of a journal, what will be maximum value on journal depression.

Naturally they are going to depend on something like length, depends on how much length we require for support, if there is any space limitation will go for more number of bearings and of course, this length also is going to play a role in load carrying capacity, larger length, larger will be the load carrying capacity. So, load capacity, space limitation if there is any space limitation that should be the first constraints.

In all that, available space is only 5 mm, designing bearing for 25 mm and also dimension 25 mm, naturally that bearing is not going to get fitted there, we have to reduce dimension. So, that space restriction should be the first point to consider and subsequently, load capacity and other reason other parameters

Coming to the clearance, naturally again first thing is that, what is the manufacturing capabilities, if you are not able to keep well with the good tolerance on the surfaces, tight tolerance on surfaces, then clearance need to be thought in some range, but we require some understanding you say increase in clearance is going to reduce load carrying capacity, larger the clearance, lesser will be load carrying capacity. Again, if there is a larger clearance, film

thickness will be increasing maximum value of film thickness is going to increase which is the function of clearance.

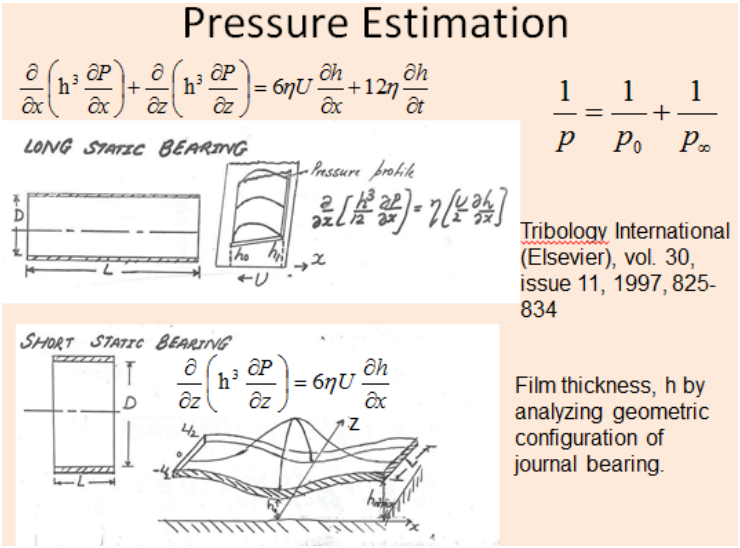
Naturally, when that is going to increase, flow rate is going to increase in cubic form, because flow rate depends on h or we say cube of h, sometime we talk about clearance related to the surface complaints, some surfaces can elastically deformed it was mentioned in rolling element bearing, when the shaft surface or the bearing surface gets displaced from its original position, film thickness is going to vary is going to change and that is instead of thinking of film thickness point of view you will say that, clearance is going to change we keep very low clearance, load is applied and the bearing is made up of the sheet metal, naturally the sheet metal will be displaced and that is going to increase the clearance or you may say you can keep a constant clearance.

Last one is the pressure you say that, pressure will be depending on the geometry of a lubricant and operating parameter. So, this is a most complex depending on the number of things in all operating parameters, operating conditions see it cannot be decided initially we require some analysis for this purpose

However, we have some table available say that, maximum pressure which need to be generated, which generates within the bearing should not exceed this limits, which are different for different materials. You are saying lead base materials are generally soften materials, they cannot tolerate maximum pressure greater than 5.6 mega pascal.

Anyway we require a larger load carrying capacity we can come to the copper based material, which has maximum pressure limit upto 28 mega pascal, again we are talking about some range we are not talking about the constant value, because the lead based material will not be only one material, it will be based on number of combinations, different attitudes can be used, different other materials can be used that is why these materials generally given the pressure limits are given in the range, they are not a single one it is a lead based material that does not mean that, is the only one material, it can be a group of materials that is why we are defining some range for that group.

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Now, we say that most critical is that, how to find out the pressure once I know the pressure distribution, then only I can find out, what will be the maximum pressure, that is why, start with again Reynolds equation, which involves the pressure distribution term left hand side, pressure, right hand side, source term.

If we assume a dealing with steady state condition I will say that, delta h delta t is 0, film thickness is not going to change with time, naturally this term also 0 and we are going to discuss only the steady state condition and this is 0, only this terms are going to act. Again, this partial differential equation will be slightly complex to solve we require finite difference or finite element method to solve it, for general classroom conditions or classroom environment, we prefer some approximation one approximation, which we discussed in previous lecture is study covered short static bearing or short bearing.

And this length is much smaller than diameter, it can be 0.25 times of diameter, 0.3 times of the diameter, which is a more dominating feature or you say them dimension particular in automotive industry.

Now, when we say length is negligible compared to diameter, naturally this term will be much more dominating compared to this first term. So, that is why for simplicity first term can be neglected or we can write this equation, second term and right hand side first term. So, this equation can be written over here.

There is another possibility, if the bearing is long, first slide of this lecture I showed a long bearing, in this situation length is much more much greater than diameter and that is why I

mentioned L/D ratio on the same bearing is equal to 4, for this kind of approximation this kind of assumption, naturally second term is not going to play major role, the first term is going to play major role that is why the relation will be modified like this.

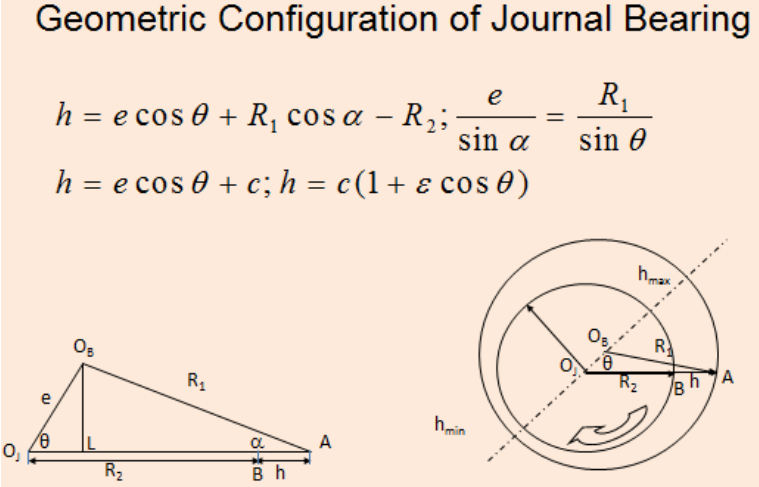
And we discussed earlier, what will be the hybridization now first we know two result this two approximation gives the extreme results and pressure estimated by either of this will not be that high in actual case that is why we adapted hybrid approach, harmonic combination, inverse combination of these two pressure here P naught or P_o is short bearing approximation or we say the length is equal to 0, so that is why the P naught.

For second approximation, say length is infinitely long that is what the P infinity and this is the actual case. So, this is the short bearing, long bearing use a harmonic combination to calculate this. If we you have some interest in doing the hydrodynamic calculation based on this approximation, you can refer this paper published in 1997 its almost nine page paper detailed paper, how to design hydrodynamic bearing accurately in simpler way, obtaining both the thing simpler way and accuracy this are the simpler approximation, but they are not going to give reliable results.

For classroom will be following the short bearing approximation, but same procedure can be used with hybrid approach also, I am going to demonstrate in present lecture, short bearing approximation and leaving hybrid approximation for you can refer this paper if you have difficulty in solving this equation, this is a detailed one. Now, what is missing in this we are talking about the Reynolds equation and which is depending on the film thickness, depending on the viscosity, depending on the velocity?

Velocity is input parameter, viscosity is input parameter, but will be changed with a temperature. So, this is most critical parameter as sensitivity is very high, the film thickness expression is important for us. So, let us concentrate how to derive expression for the h, see film thickness h can be derived by analyzing the geometric configuration of the journal bearing.

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So, next slide is completely on the film thickness and please see this and the figure we say that, there is a maximum film thickness, minimum film thickness, this is the line of centre with the centre's are going to lie on this line, O_B is a bearing centre, O_J is a journal centre, R_1 is the radius of bearing surface, R_2 is radius of shaft surface here. We are talking about the bearing bow, R_1 and we are talking about the solid shaft, hallow study when outer radius is R_2 and separation between these two surfaces will be represented in the radial direction as a h that is we are dealing with a polar coordinates.

Now, expand this or say, when we say analyze all these uneven portions for that purpose we are drawing this sketch O B, O J distance is e eccentricity, then OJB, OJB is the radius, the shaft surface is starting from O_J to the B the shaft surfaces, then OBA is a bearing surface or bearing ranges, separation between A and B is equal to h assuming and we are talking about may be theta is 0 and maximum film thickness fine or on this line and theta will be pi, when h minimum and we are going to discuss about the half bearing; that means, bearing which pressure zone starts from theta is equal to 0 and extend upto theta is equal to pi, a more meaningful results can be achieved based on this theta is equal to 0 to theta is equal to pi in this domain only.

Assuming this domain has mostly cavitation or a cavitator streamers, ratio generation will not be significant in this situation.

So, how to define the film thickness h simple approximation will say resolve, take a component of e, e cos θ or OJL as a e cos θ, OBA is a cos α, R_1 Cos α minus this R_2 that is going to give h, what we are doing we are taking component in horizontal direction or on the

horizontal line, component R_1 on horizontal line. So, it will be OJL and LA that is giving complete length, OJA.

But we know OJA can be dissolved as OJB and BA we know R_2 we can find out h by equilibrium, that will turn out to be e cos θ plus R cos α minus R_2. Actually we do not know anything about α we know the θ is going to vary from 0 to π, but what are the values of α for that purpose we can use the second equation.

We know this is a triangle O_J OBA is a triangle you can use the triangle sin rules e divide by sin α, R_1 divide by sin θ that is given over here, e divide by sin θ equal to R_1 divide by sin θ and we are interested to find out the sin α, again take this at the right hand side, e divide by R_1 into sin θ will be given as a sin α. Now, some approximation see e maximum value of e is clearance and we know clearance divide by R_1 will be much smaller ratio we often prefer clearance equal to 0.1 percent of R_1 or another one 0.001, R_1.

Naturally, e by R_1 will be un negligible quantity and is been multiplied with sin theta, even if you assume the e plus R_1 are the 0.1, this α in degree will be far lesser than 1 degree is not going to vary significantly is much lesser than 1 degree or another word I can take approximation, sin alpha equal to 0 negligible, if that is an approximation, then R 1 cos α will turn out to be equal to 1.

That is the situation this whole expression will be e cos θ plus R_1 minus R_2 we know, R_1 minus R_2 is equal to clearance and that is over here, h is equal to e cos θ plus clearance. We have been mentioned in earlier cases that, we are more interested in normalization of eccentricity or we are more interested in clearance in eccentricity ratio that is epsilon and that is why we are taking this C as a common.

When you take C as common this will turn out to be 1 plus small e divide by C that is eccentricity ratio into cos θ, another word film thickness can be represented as function of clearance epsilon and cos θ, that is why we are talking about the maximum film thickness and minimum film thickness, here maximum film thickness theta is equal to 0, the maximum film thickness will turn out to be C into 1 plus epsilon and we are talking about the minimum film thickness, this θ will be π; that means, this cos θ will be minus 1 or h will be C in bracket 1 minus epsilon that is the minimum film thickness. So, this kind of representation is helpful to find out, what will be the maximum thickness, what will be the minimum film thickness.

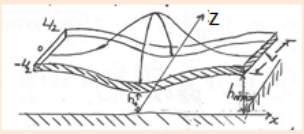
When on the film many maximum film thickness we can locate the groove at that position and we know because of that position, placement of the oil hole will be getting maximum flow rate there with the lesser supply pressure, lesser running cost. So, based on others kind of analysis we can figure out, how bearing design should be, how bearing should be designed.

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Deriving fluid film pressure considering Short Static Bearing Approximation

$$h^3 \frac{\partial}{\partial z}\left(\frac{1}{\eta}\frac{\partial P}{\partial z}\right) = 6U\frac{\partial h}{\partial x} \qquad \frac{\partial^2 P}{\partial z^2} = \frac{6U\eta}{h^3}\frac{dh}{dx}$$

$$\frac{dP}{dz} = \frac{6U\eta}{h^3}\frac{dh}{dx}z \quad \text{using symmetry condition } dP/dz = 0 \text{ at } z = 0$$



$$P = \frac{3U\eta}{h^3}\frac{dh}{dx}\left(z^2 - \frac{L^2}{4}\right) \quad \text{using } P = 0 \text{ at } z = \pm L/2$$

$$P = \frac{3U\eta}{c^2(1+\varepsilon\cos\theta)^3}\left(-\frac{\varepsilon\sin\theta}{R}\right)\left(z^2 - \frac{L^2}{4}\right)$$

Now, comes to the deriving of the pressure we say let us start for the bearing we are assuming the short bearing approximation has h is not a function of z, z can be taken out without any differentiation and again, if I assume for time being, η is not depending on the z. So, it can be taken out, it can be rearranged this equation can be rearranged like this, second derivative pressure with respect to z equal to this 6 η U this will remain constant for the one operation of course, η is going to change with the temperature we need to find the temperature corresponding change we need to change eta.

In those situations, h cube is the film thickness and the gradient of h with respect to x, this is the one and we can integrate if we can integrate once and use this boundary condition say gradient operation will be 0 at the mid plain, because we know the mid plain, there will be maximum pressure and maximum pressure can be obtained by equating the expression, gradient of pressure at equal to 0 for that position and this is shown in earlier slide.

You see when a z is equal to 0 this is the maximum going to give maximum pressure, when z is not equal to 0, this will be giving slightly lesser pressure maximum pressure lesser than maximum pressure in this situation that is the one, when you substitute this 0 at this gradient will be 0 is constant integration constant c 1 or d 1 whatever constant that will turn out to be 0; that is why it is been written simply multiply with the z in this situation you can go ahead

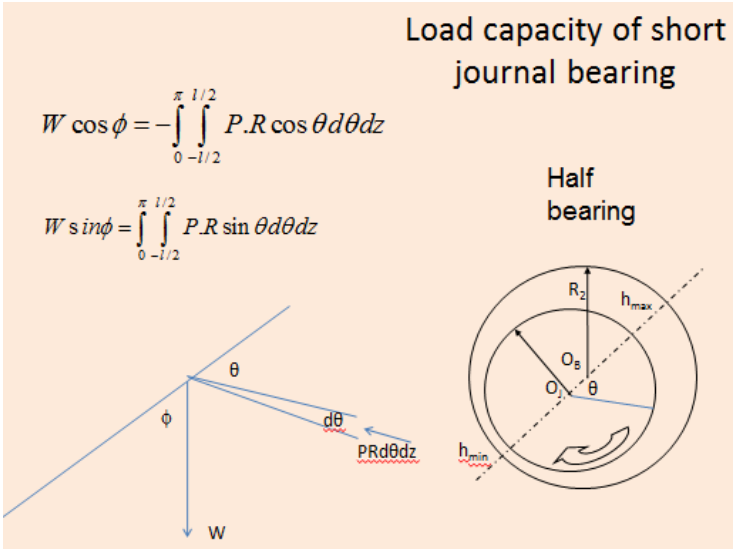
with the second derivative or second integration and you can use the boundary condition pressure will be 0 will be atmospheric or if it is in process fluid it will turn out to be some value, then we need to define for time being we are assuming the pressure, the gauge pressure outside the bearing surface is equal to 0 and that we can use any of this condition minus z equal to minus L by 2 or z equal to plus alpha 2.

We can use any condition will be getting same expression like this. So, this is the simplified expression depends on the velocity, depends on viscosity, depends on the film thickness and we have already found, what will be the h in terms of theta, naturally h need to be converted in terms of theta that will be R theta when we do that, substitute what we are getting the pressure relation or something like this.

So, 3 U velocity eta divide by this would be the c cube, but 1 c cube here and 1 c here. So, we are cancelling that c with this and that is why we are writing c square, while this is the 1 plus epsilon cos theta it representing the film thickness and this is d h by d x which is being represented like this into c which the 1 c is being cancelled with this and this is the same thing. So, this is the pressure distribution.

Now, we can find out similar kind of pressure distribution using this relation that is the pressure, what is the next point, next point for us is to find out, the load carrying capacity of the bearing which is only important aspect for us.

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For final load carrying capacity naturally we had to go ahead with the force balance equation, what is the force balance equation we say we are dealing with the polar coordinate, so better

resolve this component in polar form. We know, load will be applied on a journal and vertically down direction that is shown over here and this is the line of centre, naturally angle between line of centre and apply load will be phi.

At any angular position the angular position of theta we can take very small segment having angular extent of d theta and assuming this is the radius or. So, pressure the pressure has been generated there, force due to that pressure will turn out to be pressure into area and area we know R d theta into d z that is he what we are giving the pressure we can resolve this pressure along this line and perpendicular line now.

If we dissolve in this and perpendicular to find out the thermally to find out the force balance, naturally we need to dissolve this W also, W cos phi this way and W sin phi perpendicular, when you say W sin phi this side PRd theta dz also in this side. Naturally, summation turns out to be 0 and that is going to give force equation. So, W cos phi equal to minus, because summation equal to 0 take this term right hand side, naturally it will be minus and if the extent is a 0 to pi or a theta extent is 0 to pi we are assuming bearing is only 50 percent bearing, half of the bearing and this is known as the half sum condition.

When we know the z it is the mid plane 0 and outside is a minus l by 2 to l by 2 this integration is going to give us W cos phi, first component or one component of the force similarly, we can use the sin this is what we say the half bearing has been considered, this is the second component W sin phi equal to 0 to pi, length will be same minus l by 2 to l by 2, P R sin theta and d theta d z we need to integrate this to get the positive result to get the benefit or to find out, what will be the W.

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Load capacity of short journal bearing

$$W\cos\phi = -\int_0^{\pi}\int_{-\frac{l}{2}}^{\frac{l}{2}} p.(Rd\theta.dz).\cos\theta = -\frac{U\eta L^3}{2c^2}\frac{2\varepsilon^2}{(1-\varepsilon^2)^2}$$

$$W\sin\phi = \frac{U\eta L^3\pi}{4c^2}\frac{\varepsilon}{(1-\varepsilon^2)^{3/2}}$$

$$\Rightarrow W = \frac{U\eta L^3\pi}{4c^2}\frac{\varepsilon}{(1-\varepsilon^2)^2}\left\{\left(\frac{16}{\pi^2}-1\right)\varepsilon^2+1\right\}^{1/2}$$

$$\tan\phi = \frac{W_r}{W_\theta} \Rightarrow \tan\phi = \frac{\pi}{4}\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}$$

What we can do in simple manner, integrate it, I am not involving all the steps, but I am giving the final results of this. So, when we integrate it, what we are going to get minus U eta L cube divided by 2 c square 2 epsilon square 1 minus epsilon square whole square of this that is the first component of the load, this is the second component of the load.

Now, we can take a square root of this component and this component and find out what will be the W and that W turn out to be U that is the velocity, eta viscosity, length and it shows the length is very dominating feature in this case, larger length two times length is going to increase the load by 8 times pi and is the clearance ratio and epsilon terms. So, it is complete epsilon terms that is going to give us, what will be the load carrying capacity for given bearing geometry and location of shaft surface or journal surface.

There is an another one important we say the for location purpose we required eccentricity and attitude angle, that attitude angle can give like this the tan phi that means, the W sin phi divide by W cos phi, this term will be a numerator, this term in be a denominator when we do that, for final calculation this will be negative of this. So, final results will cut out to be something like this say, tan phi is equal to pi by 4 square root of 1 minus epsilon square divided by epsilon. So, this is going to give us attitude angle. Will continue with this in our next lecture, which will also be lecture on a journal bearing. Thanks for your attention.