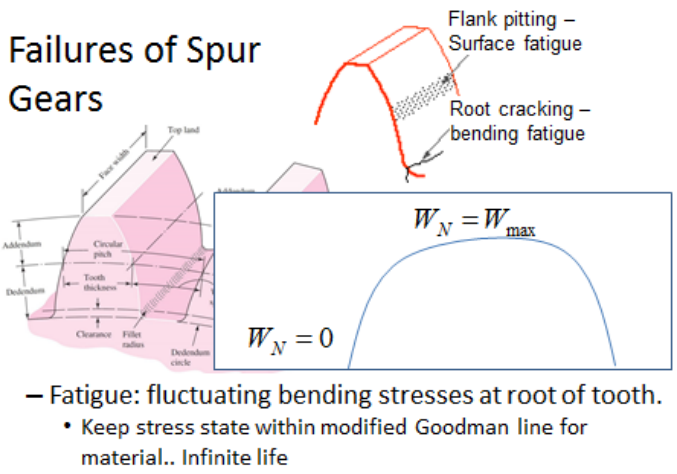
Tribology

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Module No. #06
Lecture No. #38
Surface Fatigue of Spur Gears

Welcome to 38th lecture of video course on Tribology, topic of the present lecture is Surface Fatigue of Spur Gears. Different elements are involved in the surface fatigue and spur gears. Spur gears have been already described in detail in previous three lectures. Fatigue is a failure where the repetitive loading happens or occurs on component surface and we are talking about the surface fatigue, which is more pertinent to the tribology. Mainly fatigue is more on mechanical side, where the bulk properties are used; while in surface fatigue, we use only the surface properties, we do not use a bulk property.

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So, let us start this lecture on surface fatigue, we demonstrated that surface fatigue in this case, we are treating as a failure phenomenon or failure mechanism, there are two most common failure mechanisms, we say the root cracking, that is the bending fatigue happens due to the tensile force, tensile stresses.

The other one is a flank pitting, that is a surface fatigue happens because of the compressor loading, excessive compressor loading. So, there is a different phenomenon, there is a different mechanism, again root bending may be spontaneous phenomena, when the crack is

developed and propagated. Ssurface fatigue is a slow process, it gives enough time to replace the gear component or we say gear part. However, we use the word flank over here, I have described in the previous lecture what is the meaning of flank? let me repeat it.

See, when we are thinking about invalid profile, in invalid profile, there will be hypothetical circle, a construction circle known as a pitch circle. Above the pitch circle, whatever the material for the gear occurs, that is known as the face and this is a top line. So, this portion will be known face, where the initial engagement of gear happens while this portion below the pitch circle, but above the pitch circle known, this portion will be known as the flank.

And what we are discussing over here is a flank fitting, most dominating feature. It does not happen exactly on the pitch circle, but just below on the pitch circle. When the sliding is also there, very high load is also there and they combine to create small pits, small dimples on the gear tooth and those dimples create irregularity in the surface cause some sort of vibrations, some noise.

However, the first one is a root cracking or bending; it happens, however we say in both the cases, we are assuming the word fatigue, it happens because of the loading and unloading of gear tooth.

If I develop the gear tooth surface and we say whole pitch circle diameter and on that, number of teeth and show only portion of that, what we say that, in one cycle, there may be a 24 teeth, so I can develop 24 different domains and out of 24 domains, only one domain will be there where gear tooth is going to get loaded, that is over here. See, load initially is 0, it resists to the maximum value and gradually comes back to 0 values, initially gradually and spontaneously, it comes to the 0 value. However, in another word, this gear tooth is going to be subjected to load 0, initially, then to the maximum value W max, then again back to this 0.

This is what, we say the compressive loading and fluctuating compressive loading, it is going to get subjected. However, because of the this loading, what is going to happen, there will be some component and that is going to create some sort of tensile stress at the root, that is going to cause bending fatigue; in another word, bending fatigue happens because of the tension, because of the tensile stresses.

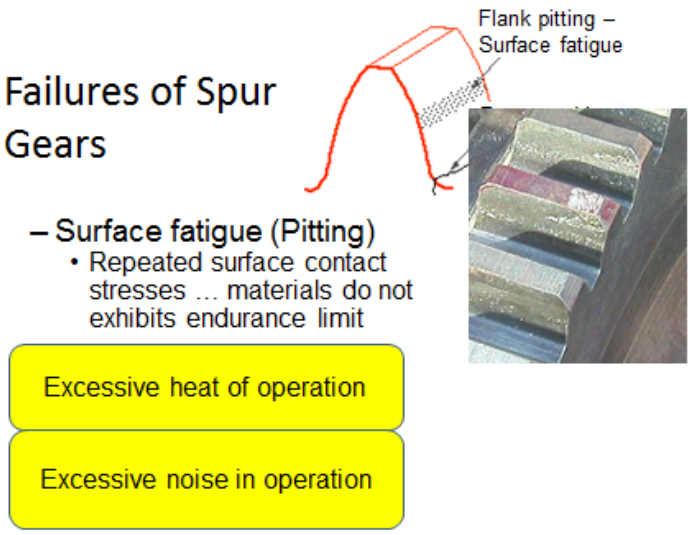
While surface fatigue, where there will be direct load on this, there will not be movement, there will not be movement over that, and that happens, because of that there is a compressive or excessive normal force that cause surface fatigue. Of course, when we are talking about

only the force, it is not appropriate, because fatigue happens because of the material properties also, it is subjected to the material properties, and it is subjected to the geometry and subjected to the load.

So, load alone will not be able to find or estimate surface fatigue, we require other parameters to estimate the surface fatigue barrier. We can see the fatigue is fluctuating bending stresses, or it causes by fluctuating bending stresses. At the root of the tooth, this is the tensile stress.

To keep this safe, without any crack generation at the root, what we need to do, keep stress state within the modified Goodman line. So, popular for your criteria line, when we say that, we need to keep stresses within Goodman line criteria or modified Goodman criteria, where the yielding as well as endurance, both have been accounted. We are not going to discuss this in our course on tribology, because it does not pertain to this topic. We will be discussing only surface fatigue value, we already discussed in previous lecture on scuffing, which is pertinent to tribology course. Now, we are going to discuss about the surface fatigue.

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So, this is a thing, we are talking about the values and this is a typical picture of surface fatigue value of the gear. I can see there are number of bits for lesser or below the pitch circle radius, this portion, this portion, this has small pits (Refer Slide Time: 7:10). So, this gear is a spur gear, it is getting number of pits on flank or tooth flank, that is surface splitting failure of the gear tooth.

So, what is the problem in this, I do not find confident attachment of the gear teeth from surface, it is not going to create much problem as such it is going to give some time for us to

replace it and problems will occur if performance deteriorates, there is no physical or complete damage of the gear, but performance deterioration happens when pits are generated.

If you say that surface fatigue, it is happening again because of the repeated surface contact stresses, again the portion will be loaded and unloaded, that is why there will be repeated surface contact stresses, what is a surface contact stress? We have already described in the previous, in basic fundamental time. And we again will try to give couple of slides on that, so that we can estimate the stress and when we say the surface fatigue comes, mostly the surface properties are accounted. In those situations, materials will not show any endurance limit or what we say, material is not going to show infinite life, it is not going to survive forever, and the depth of the material is shown.

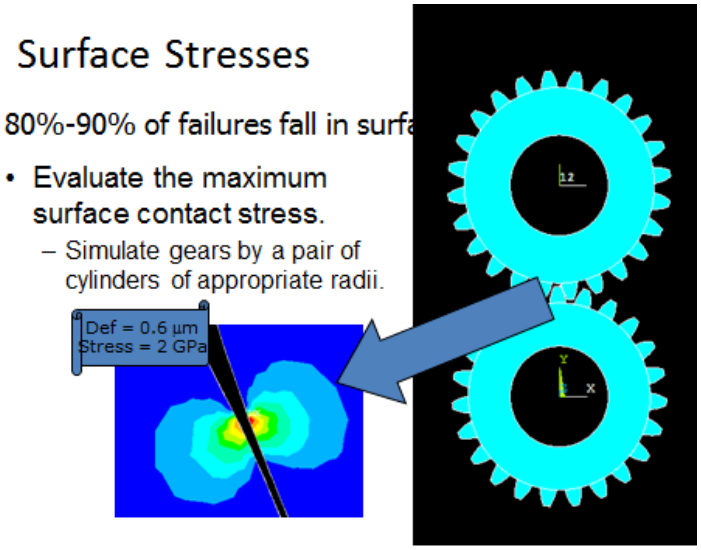
However, the two major problems related to the pits generation or generation of pit is excessive heat of operation, question comes how come the pits are generating excessive heat? They should be acting like coolant agent, they can store the lubricant, they can reduce the heat generation or we say dissipate heat much faster, but we know very well whenever there is a pit, rolling motion will reduce, sliding will increase or we say slide to roll ratio will increase.

And whenever there is a slide to roll ratio increase, there will be say higher side, the frictional heat generation will increase and that make create excessive heat. So, we have to see overall thermal equilibrium, but because of the pit generation, there will be more and more heat generation.

Now, we know there is no continuous phenomena, this is we are not going to get continuous contact, if there are number of pits. Obviously, gears are not going to get a contact, constant contact or continuous contact, naturally there will be noise generation. Question comes; do gears work without any noise? No, gears will always work with the noise, they will always generate some noise, but the noise may not be noticeable initially, because environmental noise will be much higher than the generation of noise by the gears.

But after generation of pits, discontinuity after sudden change in stiffness, that is going to give higher noise, loud noise which is, which can be heard easily and we say that gear box is not showing this performance, it is not giving its performance, we need to replace the gear box and this way we have, we know the number of pits generated on the surface, generating excessive noise and we need to replace this gears.

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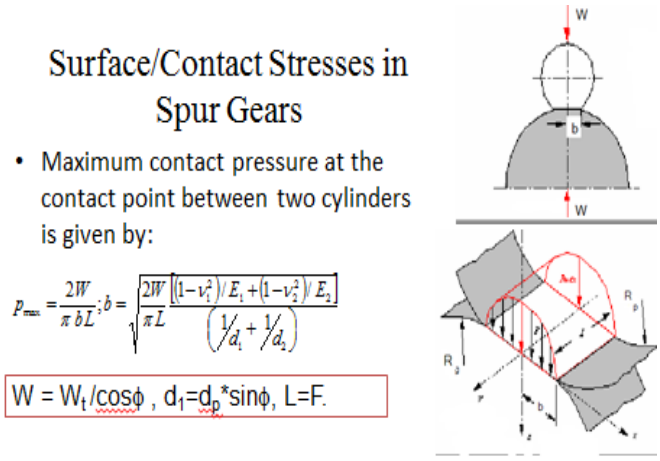
Now, when we were discussing about the bending value and surface value, how to say that tribology plays more important role compared to the mechanical engineering, as a bending value accounts to 10 to 20 percent of overall gear failure, why? Surface stresses, surface fatigue counts almost 80 to 90 percent of values of the gears.

So, tribology is more important for the gear design compared to the mechanical engineering design or we say whenever we treat the gear topic, we should use knowledge of the tribology, knowledge of the lubrication, and knowledge of the contact surfaces to get proper life, proper performance from the gears. Now, how to evaluate, how to analyze this kind of surface fatigue value, you say evaluate the maximum surface contact stress, we need to estimate what will be the maximum contact stress, difficult to estimate analytically, but we can do some sort of simplification.

How to do that simplification, you can say similar to the gears by a pair of cylinders of appropriate radius, if we know radius of the cylinder, length is known to us and the tooth length is coming into contact to forming, we can find out what will be the elastic deformation, what will be the contact stresses there? This has been already dealt, you say what we are trying to find is something like this, if this is contact pair and subjected to the load, now we are getting very high stresses here, and we need to simulate this easiest way.

Of course, we can do the finite element modeling the way it has been done here, we can estimate what will be the maximum stress; while in this case, it has been shown stress is 2 gigapascal, very high stress and deformation or local deformation of the surface is roughly 0.6 micron, very low deformation, that is why we require very good surface finish of the gear tooth.

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And to estimate this, the easiest way is to simulate with two cylinders subjected to load, we know this has high excess rate or high contact stresses. Naturally, there will be some sort of elastic deformation and that elastic deformation is represented with b, there will be matrix, overall contact patch will be 2b and it is elliptical contact, we are assuming there is a uniform distribution of stress over the length of the cylinder; that means, load divide by the length is been considered.

Otherwise, if it is other than we are talking about, in this case rectangle, this will be elliptical, may be the edges can be left when there is no contact. However, if we use two phases, then the contact patch will be circular. While in this case, we are saying that it will be elliptical, if we disregard, if we do not count the corners; in those situations, then we can assume there is a uniform distribution. So, there is, this figure shows the load is applied on the one cylinder, smaller cylinder, I can assume this as a pinion.

And one which is resting against spur gear, larger diameter and contact patch is overall 2b into L will be the, I am assuming the rectangular area, one side is a b and this is a pressure generation. We have seen that there will be parabolic ratio distribution, hence pressure will be 0 at the mid of the centre or we say above the axis of symmetry, the pressure will be maximum.

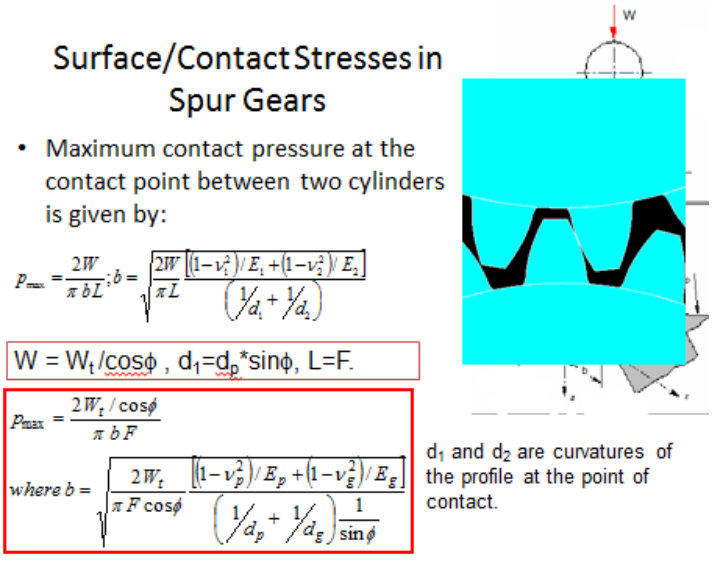
That can be given if we know, what is the radius of the pinion, what is the radius of gear. Again this is, these readings are not the same as what we discussed about the pitch circle radius, this radius are slightly different than pitch circle radius, reason being we are talking about the curvature effect of the gear tooth in valid profile of the gear tooth which may come when we make a contact. Now, ray will be different or radius of the particular contact surface, load will be different. So, let us start with the same relation what we earlier derived

in earlier lectures, we say that P maximum can be given in terms of load, in terms of contact patch and in terms of length.

When we say we are assuming as uniform distribution, that is why we are accounting W by L and this is after integration, we can find out, say this, it is depending on the contact patch. And contact patch can be given after, doing the couple of derivations in terms of again W by L, in terms of geometry, in terms of material parameters. These are young's modulus E_1 and E_2, E_1 is a pinion, E_2 is a gear, means material Poisson ratio μ_1 is a gear related μ_2 is, μ_1 is related to pinion and μ_2 is related to the gear, d_1 is you say the contact radius, it is different than the circle area or we say that, this a diameter.

So, contact diameter of the surface 1, surface 2 or pinion and gear and there will be different than the circle diameters. Again in gear terminology, whatever we have done till now, we are using slightly different nomenclature; you see instead of using L, we have been using F that is a face width.

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So, we use that and this is a, what diagram also shows where we are not talking about the like this, but there is a curvature and there will be some centers on their, we are talking about only this. We are not talking from the centre, connecting center to the, this we are talking; if I say instantaneous centers somewhere it will lie over here. So, this much distance, similarly for other pair, this much distance, those radius or those diameter will be accounted for this fluctuation and that can be given, W can be given if you know the W_t or force on the gear tooth and pressure angle d_1 in this case has been shown as product of this circle diameter into sine of pressure angle, and L what we say the length of the cylinder has been nomenclature as F face width of gear tooth.

So, we can use this kind of nomenclature and to make it same, what we have done in the gears and there should not be any confusions, we can try under, this is the guess clearly that d_1 and d_2 are the curvatures of the profile at the contact points, what is the contact point, instantaneous radius or instantaneous points of the contact and I can find out what will be the diameter, what will be the radius over there and that can be determined by using pressure angle.

And we know very well, pressure angle will keep changing, this is a nominal pressure angle and mostly counted at the pitch circle point, what we say that point where the two circles are contacting; one, this is going to change the base, this will be 0 and it will slowly increase, reach to maximum value where the pitch point occurs or with the contact point occurs.

We substitute this values, make sure that substitute W_t by cos ϕ in this relation, so what we get? $2W_t$ divide by cos ϕ divided by pi into contact patch or half of the contact patch into F is replaced there here it will L has been replaced with F.

Coming to the second expression for the b, b is been given as a $2W_t$ divide by a π, F; F is again a face width cos ϕ and this is a material constant, this is the geometry constant and again we are able to see the sine π over here, what is interesting? b is in denominator W is a numerator, L or is a F is in denominator. Now, b also contains similar kind of a, and it is W_t F. So, W_t is in numerator and F is in denominator. So, when you substitute it, turn out to be, everything will turn out to be in a square root.

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$$p_{max}^2 = \frac{2W_t}{\pi F \sin\phi \cos\phi}\left[\frac{\left(\frac{d_p + d_g}{d_p\, d_g}\right)}{(1-\nu_p^2)/E_p + (1-\nu_g^2)/E_g}\right]$$

Surface contact compressive stress

$$\sigma_c^2 = \frac{2W_t}{\pi F \sin\phi \cos\phi}\left[\frac{\left(\frac{d_p + d_g}{d_p\, d_g}\right)}{(1-\nu_p^2)/E_p + (1-\nu_g^2)/E_g}\right]$$

$$\Rightarrow \sigma_c^2 = \frac{W_t}{F d_p}\left[\frac{1}{\pi\left[(1-\nu_p^2)/E_p + (1-\nu_g^2)/E_g\right]}\right]\left[\frac{2}{\sin\phi\cos\phi}\frac{d_g + d_p}{d_g}\right]$$

$$Let \quad C_P = \sqrt{\frac{1}{\pi\left[\left(\frac{1-\nu_p^2}{E_P}\right) + \left(\frac{1-\nu_g^2}{E_g}\right)\right]}} \qquad I = \frac{\sin\phi\cos\phi}{2}\frac{Z_g}{Z_p + Z_g}$$

$$\Rightarrow \sigma_c = C_P\sqrt{\frac{W_t}{F I d_p}}$$

So, we can square it or we can say P maximum square can be given as a $2W_t$ divided by pi, this is a face width into sine of pressure angle, cos of pressure angle, geometry related parameters and material related parameters.

Now, this P maximum as we are talking about the surfaces, we are not talking below the surface. So, in those situations, maximum contact happen at the surface and then maximum contact stress, if we represent as a sigma c. So, this expression can be represented as sigma c square is equal to the $2W_t$ divided by π F sine of pressure angle, cos of pressure angle, geometry parameters and this is material parameters. So, we can find out using this expression, what will be contact stress, what will be the maximum pressure, what will be the maximum pressure at the contact point that is a contact stress and that is going to give us.

Life of the gear, that is going to tell us whether gear is going to fail or not, whether pits will be generated or not, or pits are going to generate after 1000 hour, 2000 hours, 3000 hours or maybe, they have a very long life which is beyond a life of machine or beyond the life of technology, that is in the situation.

So, we can rearrange, for our comfort or we say for our own convenience. So, we are rearranging it in this case, we said W_t by F and d_p, this three things we are keeping outside the bracket, W_t F by d_p. Now, in this bracket what we are doing, we are trying to keep all the material related parameters; in second brackets, we are trying to keep all the geometry related parameters. So, this is a geometry related parameter, this a material related parameter and we are using Poisson ratio as well as young's modulus; in this case, we are using pressure angle as well as we are using the pit circle diameters.

Now, we know this the pit circle diameters can be given as a number of teeth into module and the module will remain same for gear and pinion, that is right, when we talk about the gear and pinion, this pit circle diameter can be represented as m is a module into Z_g, similarly like this, m into Z_g plus m into Z_p, m will be common and will be nullified will be cancelled out and this quantity can be represented in terms of number of gears.

So, you see that is a, what we say Z_p plus Z_g divided by Z_g, and this is inverse of this we will say I is been represented as sine of pressure angle, cos of pressure angle divided by 2, let us say inverse of this and d g is been replaced with a Z_g d_p has been replaced with the Z_p and here d_g is been replaced with Z_g. So, this is a geometry parameter can be calculated

separately, I am talking about the material parameters material parameters has been given as a, it is a function of Poisson ratio and young's modulus.

So, once we have simplified it, our overall expression can be simplified. So, what is the advantage of this, when we do the iterations, when we say that the different pressure angle what is going to happen, this parameter is not going to change at all, face width is not going to change at all, W_t is not going to change, d_p is not going to change, if there is a different point, we can change phi and we try to find out what will be the maximum, minimum value for that. That is the way we can separate, we can do some sort of a parametric study on that or we say if you want to keep the similar kind of pressure angle, when we want to keep it as a function of d_p, we can play with them or as a function of face width, again we can find out the results accordingly. So, this kind of a formulation is helping us to estimate stress level for different parameter or with different level of the parameters.

So, this is a simplified expression which is the sigma c contact stress, a maximum contact stress is given as C_p, square root of W_t, that is the tensional load on the gear surface divided by F, divided by I and divided by d_p pressure, I will say that pit circle diameter of the pinion. So, once we know this, we can find out what will be the maximum contact stress.

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AGMA pitting resistance equation $\sigma_c = C_P \sqrt{\frac{W_t}{F\, I\, d_p} C_a\, C_m\, C_v}$

	Driven Machines				Load distribution factor C_m	
Power Source	Uniform	Light shock	Moderate shock	Heavy shock	Face width, mm	C_m
	Application factor, C_a					
Uniform (Electric motor, turbine)	1.00	1.25	1.50	1.75	< 50	1.6
Light shock (Multicylinder)	1.20	1.40	1.75	2.25	150	1.7
Moderate shock	1.30	1.70	2.00	2.75	250	1.8
					>500	2.0

$C_v = \left(\frac{A + \sqrt{200V}}{A} \right)^B$ where $A = 50 + 56(1-B)$; $B = 0.25(12 - Q_v)^{2/3}$

Now, we have derived the equation, we have found out what will be the contact stress, what will be the maximum contact stress. But AGMA is a American Gear Manufacturing Association is not happy with this equation, we know this equation does not involve velocity related parameter, lesser velocity or larger velocity, relations are the same.

It does not involve effect; it does not involve kind of the shock loading, impact loading. So, that is why they included three more parameter, C_a, C_m, C_v, velocity is related, this is face width related, this is application related, naturally these parameters are going to increase as stress level.

Coming to the C_a to the first one, we say that we need to find out what is a driving machine, where this sphere is going to be decide what kind of application we are using so that when power source or driving machine is uniform, they can take an example of electric motor, then and of course, the machine which is driven is also very simple and uniform when shaft simply rotating, then this form, this parameter will be one but they are subject to a light shock or heavy shock, this parameter is going to be more than one, for the light shock, some change in there is some sort of discontinuity in the force, discontinuity in transformation, then this will be 1.25 for the light shock, for the heavy shock it will be 1.75.

Coming to the extreme case, this is a moderate shock; in this case, it may take an example of single cylinder engine, we know this spark happens once in every cylinder and once in each cycle. To almost 270 degrees, there is not much force; not much pressure generation. So, in that case, there will be some sort of change in loading and unloading that is going to give more over impact loading. So, in this satiation and if there is a moderate shock and vehicle is driven on the road, we can take this factor as 2.0 for a single cylinder and in the road, wheel is moving on the road.

What we know very well, the roads will not be perfectly smooth, there will not be having complete carpet, there will be some uneven surfaces. So, that is why there will be moderate shock on that, and we are taking this factor as a 2.0. So, depends on a situation, depends on application, we can choose this parameter, it may be lower parameter or more than sudden value this parameters. Of course, these are the initial designs, so once we do it, finally we need to do experiment and then that is a situation, we can change this parameter as per the real conditions. So, a C_a can be figured out using this table, and this is initial required from design point of view.

Then comes to the C_m, C_m is more like a face width parameter and gives whether the load is evenly distributed or not. We know larger length will not be able to distribute load evenly, uniformly, there is a possibility of some sort of deviation in linear profile, and there will be some sort of misalignment and misalignment that is going to introduce additional load. That is why, we never take complete length, if the face width is a 10 mm, we will be taking may

be say maximum 62.5 percent of that, upto 6.25 mm of that length, that is basal thumb rule, it is not 100 percent correct, or not 100 percent scientific. But has been observed, whenever we take this length is mostly maximum, value is roughly 62.5 percent and that is why we say that, if their face width is lesser than 50 mm, what we take this C_m of this 1.6. If I take the reverse of, the inverse of the 1.6, it will turn out to 0.625, that is what I was mentioning a 62.5 percent of that. We do not count on whole the face width; we count only the 62.5 mm of 62.5 percent of that face width.

Even this, again a discontinued bands if this is lesser than 50 mm, if it is the length or face width is more than 50 mm, and lesser than 150 mm, then this factor is likely more than 1.6, there is a slightly more penalty on this.

Of course, the maximum penalty comes on when the face width is more than 500, here we are counting only 50 percent of that. We are not counting 100 percent, we are just counting just 50 percent on the face width, of 50 percent face width is being used as a, so that adjustment factor for misalignment or which has a misalignment, there will be excess of loading and excess of loading that is, can be accommodated, because of the excess of length, we will be counting only 50 percent of that length, excuse me.

So, in that way, we are able to account what will be the misalignment and then comes the dynamic factor, velocity factor. Interestingly, this is one of the complex factor, it is not as easy as we see to be figured out it depends on number of parameter say that they are active constants or the two variables, one is A and the other one is B. Apart from the velocity, here the V is a velocity, so V is a velocity can be accounted or C_v factor can be accounted based on that, but or in other words, if V is 0, this factor will turn out to be one.

If V is higher, naturally this factor will be more than 1, and this is also depending on the B and there is also A and B are not independent, they are dependent on each other, so that A is a function of B. So, it is important, when B is equal to 1, then which is B is equal to 1, that will turn out to be same, here it will be the 50 by 50, the whole velocity parameter will be accounted or A and B will not be accounted in that time, because of the value of B, however B can be represented in terms of the quality parameter.

So, it is highly complex, it is not as it stood forward as C_a and C_v depends on A and B, A depends on B, and B depends on the Q_v, that is the quality parameter and this quality parameter depends on the surface roughness of the gear.

So, indirectly what we are doing, we are trying to involve the film thickness parameter in this, we are trying to involve what will be the EHL conditions, we know very well if the surface roughness is a higher, then the film thickness will not be able to sustain, there will be bonded lubrication or lower hand of the mix-lubrication. And that is a not going to give a very good co efficient of friction, it will be having high co efficient of friction, it is not going to give good results to us, and that been indirectly we finalized by using the k_v, k_v a factor.

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Calculation of Factor C_v

$$C_v = \left(\frac{A + \sqrt{200V}}{A} \right)^B$$

$$A = 50 + 56(1 - B) \quad \text{and} \quad B = 0.25(12 - Q_v)^{2/3}$$

AGMA Q_v	Tolerance	C_v for 16.86 m/s velocity
9	15 µm	1.34
10	10 µm	1.23
11	7 µm	1.13
12	5 µm	1

And Q_v is given as that, if we know the quality of the gear, generally we give a quality from 1 to 12 is a cost and we have a quality of 5 and 6. When we talk about automobile engineering or automobile vehicles, generally we keep this Q_v higher factor, that Q_v is a 9, for ordinary case, we use in may be say, motorbikes and in cars. But on higher end, if you want more and more smooth flow, we want noise free car, does not make much noise, because the gear box, we need to go for high quality, the high quality is difficult to achieve, requires extra manufacturing process.

So, this is over all gear will turn on costlier, but this can be quantified based on the tolerance levels, say that for gear quality of 12, equal to 12, this tolerance range is a 5 micron; that means, deviation can be maximum 5 micron.

While the quality and 9, deviation is three times higher, and is the 15 micron. So, surfaces can deviate to 15 micron, there is not much problem or it is permissible in quality. Naturally, we go for the lesser number of, the quality parameter, this deviation will increase, we will be having more flexibility in tolerance, lesser manufacturing problems, but running time is having to generate a dynamic load. And you know very well, when we use this parameter, there will be variation in the factor, so that is giving you here.

So, when the quality is 12, assuming at the maximum best quality, and deviation is 5 micron, in this situation, this will turn not to be 1, because Q_v is the 12 when you substitute over here, what is happening, 12 minus 12 is 0. So, B will turn out to be 0, B is 0 in this case, A is 50 plus 56 constant 56 divide by 56 in this case, and here it is 0 value. So, whatever, in that case, does not matter much to us, because the power is 0; when power is 0, it will always be 1.

Here, may be, 510 or 1000 or 10000 and 100000 velocity, what do the velocity do and is not going to affect, it is not going to change this C_v parameter.

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Ex: A gear pair (Z_P=23, ϕ=20°, Z_g =24, m=1.75, F=10.0 mm) transmits 8 N.m torque from crankshaft (rotational speed 8000 rpm) of single cylinder IC engine to wheels. Bore diameter of pinion is 17 mm, and bore dia of gear is 20 mm. Using AGMA pitting resistance formula, determine the maximum contact stress. Assume gears' quality = 9, E = 2.e5 MPa, ν=0.3

AGMA pitting resistance equation $\sigma_c = C_P\sqrt{\frac{W_t}{F\,I\,d_p}C_a C_m C_v}$

$C_P = \sqrt{\frac{1}{\pi\left[\left(\frac{1-\nu_p^2}{E_P}\right)+\left(\frac{1-\nu_g^2}{E_g}\right)\right]}}$ $\Rightarrow C_p$ =187 2.0 1.6 1.34

Ans: 1334 MPa

$I = \frac{\sin\phi\cos\phi}{2}\frac{Z_g}{Z_p+Z_g}$ $\Rightarrow$ I = 0.0821

AGMA Q_v	Tolerance	C_v for 16.86 m/s velocity
9	15 µm	1.34

So, based on input parameters, we can find out the C_v, we can find out Q C factor, this increase the penalty factor and C_v factor, to how to utilize this, let us take an example, see a gear pair, we know very well gears are always operated in as a pair, single gear does not have any meaning, it cannot be used, it cannot transmit, it requires other component to appear. So, the gear assembly which always comes, it is not a single gear.

So, this gear pair has a pinion and gear, and number of teeth only differ by 1, there is another important thing, we do not generally give the same number of teeth on gears, even though we require same velocity ratio, we never use, reason being same point keep coming in the contact, and the life of those gears will be lesser than if I keep one more over this, one more minus same point will not come in contact, different points will be coming in contact and there will be different pattern and we say, that more gradual, we compare to the same number of teeth gear.

Here, pressure angle or the nominal pressure angle is given as 20 degree, module is being defined as a 1.75, face width as a 10 mm, it is required to transmit 8 newton torque, from the crankshaft, rotational speed is giving and of course, the crankshaft of a single cylinder change in.

So, here it is clearly indicated, this is a driven machine or driving machine, driving machine is shaft with a shaft, only one stroke or where as the few degrees, spark will be generated, and maximum load will come at that time, remaining three cycles load will not be that dominating or I will say, there will be huge fluctuations in power generation.

Even though, we will use a ply wheel on other parameter, still the fluctuations will be on higher side, that is why we need to account high impact loading in this case, and factor C will be now treated as a moderate factor, modern level factor, or moderate shaft factor in this case. The engine is connected to the, gear box or gear pair is connected to engine wheels, again the wheels will be also experiencing some sort of unevenness on the surface, that is going to cause a moderate shaft or you say moderate to moderate shaft, this will give us a C_a equal to 2.0 and the initial label.

When we do real experiments, we can change this value, for the initiation, we will take this C_a as a 2.0, here again the both diameter or the pinion is given as 17 mm and diameter of the gear is given as the 20 mm. These parameters given to us may not be useful directly from our point.

But when you are trying to find out the pinion failure, these parameters are required to find out what will be the real factor. Now, we are saying that use AGMA pitting resistance formula; that means, we cannot use the simple constant based formula. AGMA has given modification factor along with the basic conduct equation, to those factors need to be accounted and the C_a need to be accounted, C_m need to be accounted, and C_v need to be accounted.

Determine the maximum contact stress, we are trying to figure what will be the maximum contact stress, and if possible, check it, whether it will survive or this kind of gear profile will survive or not, and if possible you estimate the life of the gear. Now, there is some assumption given to us, assume gears quality equal to 9, quality level is 9 young's modules is a 200 gigapascal or 2.2 into 10 is to 5 megapascal, Poisson ratio is a 0.3.

So, all these material parameter have been defined, if we know the quality factor we will be able to find out the C_v, we have a face within known to us, based on this, we will be able to find out what will be C_m, use those formulas or we say AGMA pitting resistance equation, sigma c can be represented as material parameter, applied load, faced with geometry parameter with certain diameter of pinion, application factor, face width factor and velocity factor. When W_t can be figured out, because in this case of power transmission is being given as a top is being given as a 8 Newton meter and we know what is a pit circle diameter, we can use a pit circle, circle radius, 8 divide by pit circle radius, that is going to give us a W_t and W_t is required over here.

Now, face width is already given as a 10 mm, we have already panelised this face width by using the C_m factor, that is the 1.6 or we are taking a factor face width F divide by 1.6, C_v can be calculated using the quality factor, we say the 9 is the quality factor, we substitute value in B, B will turn out to be giving some value and finally, A and based on the C_v. And this has the 15 micron, this records the velocity also, because we found C_v depends on velocity also, velocity can be calculated as π dn; n is being defined as 8000 rpm and π, we know as 3.14 into d_p, d_p can be figured out that 1.75 into 23, that is going to give us a d_p.

So, once we know d_p, we can find out the πdn as a velocity, once we know velocity, we know A and B factor, substitute this value the C_v will turn out to be 1.3 form. I am not deputing those calculations. because all these calculation being done in previous slides, we know the quality, substitute there will be 12 minus 9, the 3, some power that is going to give the B value, based on B, we will be able to figure out A, based on A, we will be able to figure out C_v.

So, the same calculation is being done in previous slide, which was shown somewhere here. So, quality 9 and B is here, when you substitute this 9, this 12 minus 9, the 3, power is 2 by 3 multiplied 2.5.

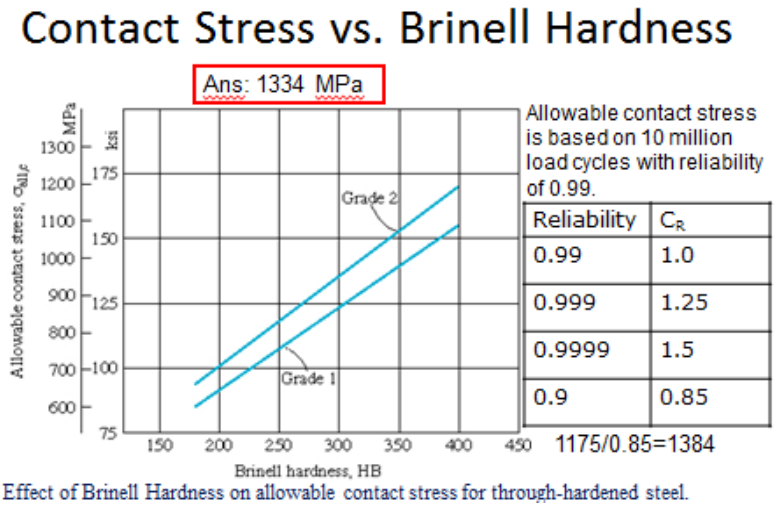
If we find out and substitute A, we will be able to figure out A, substitute this A in this equation, velocity π d and mention how to calculate for our application is turning out to be 16.86. So, use this parameter, we will be able to figure out C_v. So the C_v is known to me, or C_a figure out, C_m we know the lesser than 50 mm is the cost and the 1.6, use this value, now coming to the material parameter, the young's modulus is given and interestingly, the young's modulus for pinion and young's modulus for gear are the same, they are not changing, they remain same.

And again the Poisson ratio of pinion and Poisson ratio of gear is same. So, I do not have to calculate these brackets separately, and this brackets separately again is the same. So, I can multiply the 2 π into 1 minus new square divide by E. So, this parameter can be calculated substituting value, after finding this, we are getting this C_p is equal to 187. Coming to the geometry factor, pressure angle is known to us, that is 20 degree substitute over here, sine 5, sine of 20 degree cos of 25 degree divide by 2, number of teeth are known to us that is 24 here, pinion is also known to us; that means, 24 divide by 47 will substitute all, this value what we are getting I is equal to 0.0821.

So, I know C_p, I know C_a, I know C_m, I know C_v, we also know what will be the W_t that can be simply figured out A divide by radius, F is known to us, d_p can be calculated, based on that we can find out sigma c, contact stress is maximum contact stress, and that is standing out to be 1334 megapascal or say 1.33 gigapascal, stress level is very high.

As for the question, we have determined maximum contact stress, the point comes, will this contact stress create, generate pits on the surface, if yes, how to estimate.

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Effect of Brinell Hardness on allowable contact stress for through-hardened steel.

So, for that purpose, we require some material data, there will be number of material data, just for the completion, I am showing only one chart, when this chart says allowable contact stress and here in x axis, it is being given the Brinell hardness and this figure is for, through hardened steel, it is not for the case hardened steel, it is for the through hardened steel and hardness is being given as a Brinell hardness, Brinell hardness number.

Again, we have two grades of material, grade 2 material is better than grade 1 material, have a more precision compared to grade 1 material, or based on this, we can figure out whether the material is going to survive or not, how is the gear to going to survive or not.

And what is being mentioned as a Brinell hardness is increasing, allowable contact stress is increasing, that is we know very well as the hardness increases and more and more compressive resistance on the surface and then, we will be able to give high or will be able to sustain high contact stress and that is been shown over here.

What is the interesting thing, what we got the 1334 megapascal and this we get this table or the this graph, show the maximum value as a only 1300 megapascal; that means, I cannot use grade 2 material or grade 1 material for our application, whatever we have discussed.

So, what is the possibility, how to think over, how to utilize this kind of table, we require some additional thing, you say this allowable contact stress is for 10 million cycles, 10 million load cycles with a reliability parameter of 0.99.

If we do not require that high reliability, so we require 85 percent reliability, we require 80 percent reliability and this I can utilize this table for our purpose, what when I talk about the reliability and the shear factor related to the reliability for 0.99, this factor is 1.0.

Now, if the reliability increases, this factor also increases, in fact active stress are going to increase, when further reliability increases this is 99.99 percent, this factor is 1.5. If I want only 90 percent reliability, this factor is a lower or we say whether we are talking lesser reliability this material may survive. So, let us check it, you say here at what do the maximum value, it can sustain it is something like 1175 megapascal, maximum value, we are treating the maximum value of this grade material, which happens around 400 Brinell hardness, 400 HB, that is giving the results of 1175 megapascal.

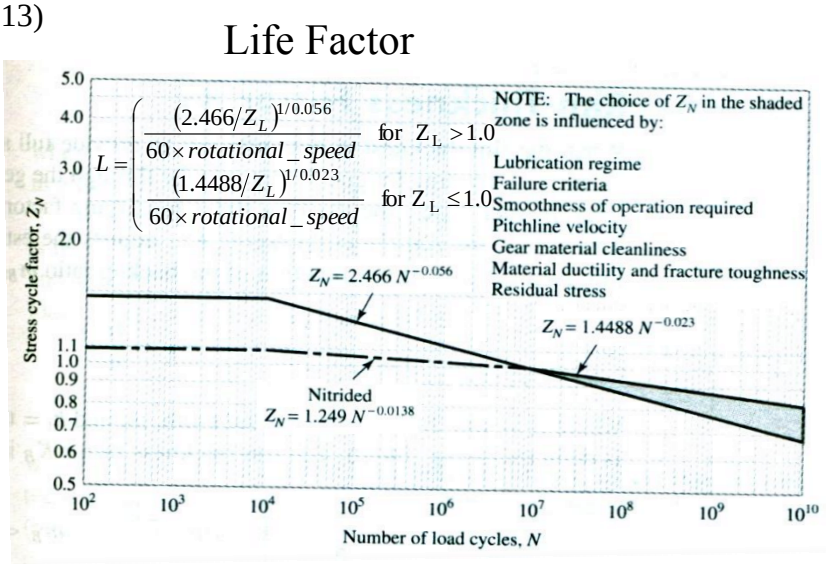
Now, I am talking about the reliability that is the 0.85, that is showing as the result 1384. So, this is going to, give us actual strength of 1384 when you compare directly with this. Or in other words, we can directly multiply 1334 into 0.85 and find out how much is stress will be generated, when we talking about the reliability ever when we talk about the allowable stress, generally we divide, we do not multiply over here, but we can do answers will not differ, answer will remain same.

So, in this case, we say that 1384, it can survive if the reliability is only 85 percent and what we have generated is stress is only 39 to 34, which is on a safer side, which is lesser than 13, which is the lesser than 1384. So, we can say this gear is going to survive with 85 percent reliability, but we talk about the 90 percent, more than 90 percent reliability, this gear may not survive and we need to go for higher hardness.

We know when we talk about the through hardness; maximum achievable hardness is roughly 400 H B. So, that is why it has been stopped over here, beyond that hardness, we generally require case hardening process, it may be night riding, may be carburization, or mixture all these two, we need to do a surface treatment and in those situation, what we are going to get, we are going to get higher hardness, but for the few micron thick layer or a few some lesser than 1 mm thick layer, after that the hardness will continuously change.

And we too analyze that, we need to go ahead with highly sophisticated analysis which is beyond the preview of this course. So, will be talking about this once we, know what is the maximum stress, which has been generated, we can find out what would be the reliability for the definite life which we are estimating and of course, in this case, we are mentioning is the 10 million cycle life.

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Now, if we want more life or we want lesser life, we can see this kind of table what you say that this is the life term, if you want the lesser life from the gear, this factor is going to be more than one; that means, this is going to give as more for the higher stress and here it is given as some value into a number of cycles, power something.

That is negative as the number of cycles increase, naturally these factors will continuously decrease. Now, here it is a coming some sort of merger or 0.10 is to 7 cycles the 10000000 cycles. So, we say and beyond that, this is completely depends on the kind of environment you are giving.

What we say, the lubrication regime for the smoothness of the operation so that what was the roughness of the regime and material parameters on residual stress, here we will not be able to discuss about this, which are more mechanical parameter, but we can think about the lubrication regimes, we can say how we can provide the better lubrication regime. So that we can experience or we can see, the higher gear life or high survival without generation on pit and whatever the stress level, it is going to give intimation, generation of first pit, it is not going to give intimation of series of pits.

Once one pit is generated, it is going to lead a generation of number of other pits, but the first pit generation, we can estimate using our formula which has been described in the present lecture. Now, if we find value of the Z_L, for some cycles turning out to be more than 1 or lesser than 1, directly from this, we can find out what will be the life of the component, I can and that is the modification factor, generally suggested by the AGMA, we can utilize this rotational s_p and we can figure out what will be the life of that component.

However, we were discussing about the lubrication regimes, with the bonded lubrication or mixed lubrication or elastohydrodynamic lubrication, there are some guidelines, how to choose what kind of velocity or viscosity should be selected for appropriate life of the gears.

(Refer Slide Time: 53:38)

Lubrication

- Splash lubrication, when power transmitted < 100 kW and Pitch vel < 10 m/s.

$$Vol(in m^3) = (0.0035 \text{ to } 0.011)\, P(\text{power in kW})\left(\frac{0.1}{Z_p} + \frac{0.03}{2+U}\right)$$

depth to which gear is immersed

h = (1 to 6) module in mm

- Pressurized lubrication (by oil jets) for large gear train transmitting 10,000 kW.

Pitch line velocity ($m\,s^{-1}$)	1	2.5	4.0	10	16	25
Kinematic viscosity (cSt)	180-300	125-200	100-160	70-100	50-80	40-65

So, we generally go ahead with the gear submersed in oil, common example is ice engine, or whether gears are generally submersed in the crankcase where the some lubricant is provided. And we say that, this kind of splash lubrication is provided for the power lesser than not 100 kilo watt, that is substantially high powered 100 kilo watts and pitch velocity also lesser side 10 milli second millimeter per seconds.

So, when we have this kind of condition, we can think about the splash lubrication, what is the splash lubrication, we say that gears are kept in some oil and because of the rotation, they have the splashing action and that will be able to give us generation or provide appropriate lubrication.

And what should be the volume of the oil which we are going to utilize here, and what should be the depth of the container in which we are using the oil, or you say that if a container is a square container or a box safe container, what there is some depth, this depth is important for us and then depth volume is important overall.

So, depth in which gears are immersed of it so that from top surface of the gear to some surface can be figure out, that h can be determined using this illustration we say 1 to 6 modulus in m m. So, with the module is around 4 mm, so we can say 1 to 6 if I take mean value initially. So, 3 into m that will 12 mm, one gear at least should be dipped in oil and it should provide this much volume so that sufficient oil is there and there is no generation, no air contained in that or splash lubrication is happening properly.

However, if the power rating is increasing, we will be requiring more and more faster cooling purpose oil with some velocity and that comes with the pressurized lubrication for the last gear, something like this, say that depends on this kind of rating, and depends on pitch line velocity, we can choose appropriate viscosity of oil.

Interesting thing is, for low velocity, we are choosing a thicker lubricating oil; for high velocity, we are choosing a thinner lubricating oil, compare here, when a pitch line velocity is a 25 meter per second, we are choosing this viscosity in centistoke 40 to 65 while pitch line velocity is very low is a 1 meter per second we are choosing very thick called as a 180 centistokes to 300 centistokes.

So, depends on the pitch line velocity, depends on the power rating we should choose the lubricating media as well as lubricant properly, and if we select this, there will be appropriate life of the gears. Again we are mentioning here, gear topic itself is a sophisticated topic, it

requires a lot of calculations, lot of iterations. However, whatever we have been presented in present course, they are good from understanding point of view and from initial design point of view, we cannot optimize this gear, based on the knowledge, given in the present lecture.

But when we talk about the optimization, we should refer to the detailed analysis, detailed finite element modeling of the gears. With this, I am trying to close the topic on the gears, we will be discussing in the next lecture or we will start next lecture with topic, journal bearing is one of the common fluid film bearings, fluid film bearing may be based on the squeezing action, it may be based on the hydrostatic action, may be based on the hydrodynamic action, we will try to cover those topics from next lecture. We have four more lectures from this course on tribology, we will be discussing in next lecture, thank you.