Tribology

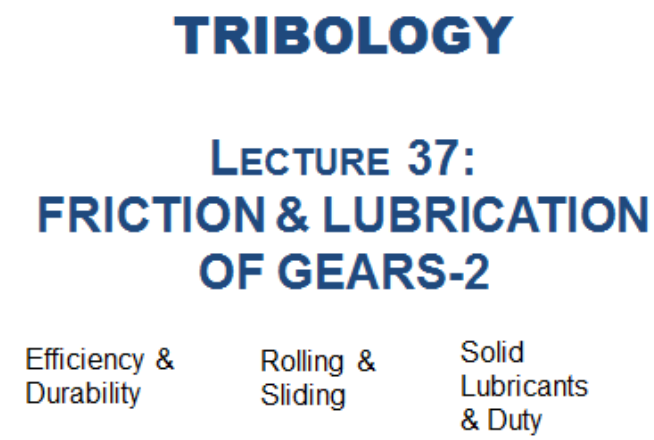
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Module No. # 06

Lecture No. # 37
Friction and Lubrication of Gears (Contd.)

Hello and welcome to 37th lecture of video course on Tribology, topic of present lecture is Friction and Lubrication of Gears, this is the second lecture in this series. In my previous lecture also we had same topic, lubrication friction and lubrication of gears. We mentioned in previous lecture, that there are two sources of friction, when we discussed about the gears.

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And those sources will decide what will be the efficiency, that is why if we do that kind of analysis, we will be able to figure out what will be the efficiency and that lubrication part is going to decide what will be the durability of the gears, what will the survivability of the gears, whether the gears are going to wear out or not. And if they are going to wear out, what will the rate of wear? That depends on lubrication, kind of lubrication mechanism we are using.

And we discussed about the rolling and sliding, we say that the pitch circle diameter particularly in spur gears, there will be rolling motion. And in rolling, mostly there will be elastohydrodynamic lubrication mechanism, because the pressure generated at the rolling contact substance is very high, substantially high, here we say more than one gigapascal. And when the pressure is more than gigapascal, naturally there will be thickening of grease or

liquid lubricant whichever we are using, that technique will convert semi solid to the almost solid and liquid to the semi solids.

In addition, there will be elastic deformation that is why we say that elastohydrodynamic lubrication mechanism. In addition, there will be sliding, when we talk about the pitch circle diameter there will be rolling, but below base circle diameter and above pitch circle diameter there will be some sort of relative sliding, relative velocity of the interface, that will be causing some sliding or friction of the surface.

So, we need to control that sliding using lubricants, then we discussed something about the solid lubricants, say we can use solid lubricant something like nylon, we can use a solid lubricant like polymers, we can make it gears directly from the polymers, then we do not require extra lubrication.

But depends whether we have application of light duty, medium duty or heavier duty, medium duty load application and heavy duty of load application, we will not be able to utilize polymers in those situations. So, polymer materials are suitable only for the light duty applications, for heavy duty we need to use some sort of liquid or semi-solid lubricants. Solid lubricant coating can be done, but it will wash out and we need to replenish one way or another way.

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Toothing Power Loss

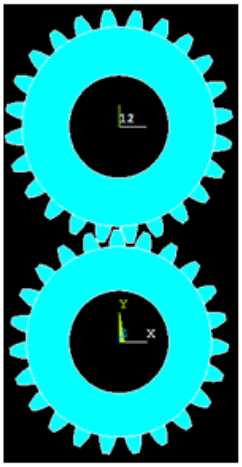
- Idle power loss
- Load power loss

$$P_{load} = F_r \, v_g$$

$$P_{load} = \mu W_N \, v_g$$

Constant or variable ??

$$\mu_m = f\left(W_N, b, V_{pitch}, \eta, R_{composite}\right)$$

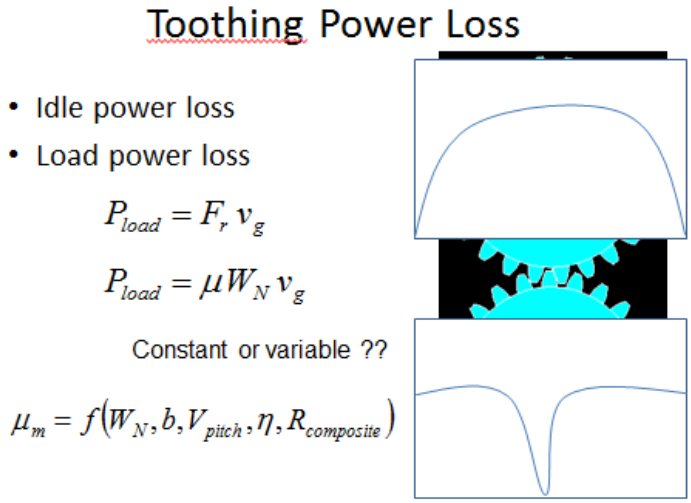


We discussed about major factor as tooth power loss, we say that tooth power los can be divided into sub categories; one is called ideal power loss, that means there is no load applied

on the gears, but still there is a speed variation or speed change that will cause churning effect.

We can see this kind of the tooth profile and we assume this whole tooth profile is submersed in liquid, what will happen? There will be some sort of churning effect, some sort of a splashing action, when the this wheel starts rotating and that is splashing is going to consume power, that will not be without power. So, that is ideal power loss and this ideal power loss will be higher when the speed is higher in other words churning loss, loss will be higher with increase in a speed, it will heavily depend on the speed, other one is a load power loss. We say that this kind of power loss will be depending on how much load we are applying; more and more load more and more power loss.

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To simulate this, what we use the relation something like this, the P loads equal to F_r into V_g, this is the sliding velocity. This requires a sliding velocity with a rolling velocity we are not considering there is a power loss, and this is the force which is being applied over other contact surface or it is a friction force and that can be represented as a cultural friction into normal force, normal force on the surface. So, P load which is required power due to the power loss, due to the load will be given as the P load equal to mu, the coefficient of friction, and normal load of the interface into sliding velocity.

We know sliding velocity will be 0 at the pitch circle diameter, so the pitch, this power loss will be 0. Other thing is that, whether this μ remains constant or not, W_N will remain constant or not V_g will remain constant or not. In actual case, μ is highly variable, will change with the

radius of curvature, it will be depending on the geometry, it will be depending on the normal load and V_g is a sliding velocity.

Now, sliding velocity itself will change continuously, it will be the maximum at extreme of gear tooth, will be 0 at the pitch circle. So, depending on what is end of phase or what is the contact point, that sliding velocity will be decided or in another words, this power loss due to load is highly variable, it is continuously changing with position to position, it will not be a constant value. So, we need to take some sort of mean value.

Now, when we talk about the mean value, naturally we need to find out the mean value of the coefficient friction also, that mean value of coefficient friction often is represented as number of variables that is a normal load. If the load is very high, there is a possibility of converting from full film lubrication to the elastohydrodynamic lubrication to the mix lubrication and to the boundary lubrication.

And we know very well, co efficient of friction will be different in all four regimes, or in other words, gear may experience also regimes, that will be dependent on the speed as well as normal load which is been applied to the gear interface. Now, when we take the gear load, naturally the phase width will also matter, because the stress which are generated, plastic deformation or elastic deformation that is generated will depend on the phase width also.

In addition, there will be rolling velocity, we say that pitch, V pitch is the rolling velocity, rolling velocity, pinion rolling velocity of gear or summation of this rolling velocity, is going to affect the mu value, then comes the viscosity eta. So, this viscosity higher the viscosity, there is a more possibility of higher value of mu m. But this mu m will depend on the range, there is a possibility that we take very thin oil almost negligible viscosity, but that is going to cause gear to operate in boundary lubrication; in that case, mu m will be higher.

So, overall we need to see the package, we need to see what is the factor, how it is depending, there is no direct relation between μ and η, but there will be indirect relation depending on the load regime, then comes surface roughness of the gear and pinion surfaces, that is why we are writing the R composite it is a surface roughness or composite surface sharpness.

Now, when we discussed about normal load in previous lecture, we mentioned that there is a contact ratio, we discuss about the contact ratio. Say, if the contact ratio is more than 1 that means some timed during the rotation teeth pair are coming into contact. And because of that, there will be change in the load quiet possible during engagement, load slowly increases,

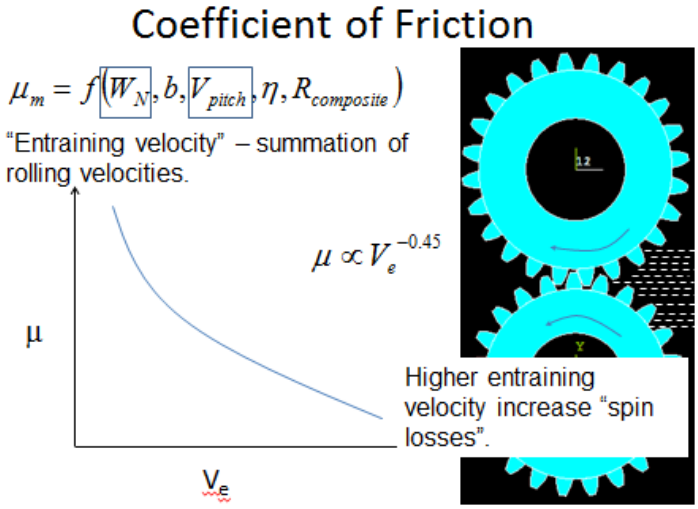
reaches to the steady value and after the time of the disengagement again, this load will start decreasing.

So, when gear tooth going to experience this kind of load variation, we need to use that kind of the load when we try to find out, what will be the coefficient of friction and another thing, there is a possibility of change in coefficient of friction itself, because when we are talking about few degree rotation of the gear wheel, we are able to see the coefficient friction itself is going to change.

It may vary little bit depending on surface roughness, depending on the other geometry, depending on material film, but it will reach to the 0 value at the pitch circle diameter, the mean value that is what the 0 value. Before that, there is a higher value and after that there will be higher value, but the pitch circle diameter this coefficient of friction is going to be 0, that is why we need to model it properly, it should come with this kind of characteristics.

May be initially, there is some sort of modulation, some sort of variation, may be statistical thing, but it will come to 0 at pitch circle diameter, when we are talking about spur gear, helical gear, of course when we talk about the high powered gear, again in those situations this velocity or this coefficient of friction will not be 0, there will be sliding and that will cause some value of coefficient of friction.

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So, we discussed about the normal load in our previous slide. Now, we are going to discuss about the rolling velocity V pitch. Now, V; this is the summation of the rolling velocity of gear and pinion, and it has been observed with high rolling velocity, coefficient of friction

will decrease. With high velocity, co efficient of friction is going to decrease, what is the reason for that? See, I am assuming these are the gear wheels, maybe I can assume this is the pinion, this is the gear and they are rotating and what will happen, and there will be some sort of tangential velocity generated at this interface.

And this having some sort of profile, see it will get some sort of liquid over here and with a rotation, dragging action of the liquid will happen, and more and more dragging action will happen when there is higher and higher velocity, or we say that this is more like a pumping action, this oil, this liquid lubricant is going to get pumped from this side to other side and that is going to generate more and more lubrication or we say that, if initially there was a boundary lubrication or lubrication; because of this pumping action, liquid will be at the interface and then it will be elastohydrodynamic lubrication or full film lubrication or mixed lubrication entirely depending on the load, how much load we are applying.

That is the why we are showing this kind of curve as the coefficient of friction as V_e which is summation of rolling velocity of gear 1, rolling velocity of gear 2 is increasing coefficient of friction is going to decrease. Of course, their final limiting value, it will not be continuously increasing and often this coefficient of friction is represented in terms of V_e, we using this relation we say that mu is proportional to V_e power minus 0.45 that is the inverse relation, we say that this is advantageous, this figure shows higher and higher V_e will be better and better results for us, better and better efficiency, but this is what we are talking about the load (Refer Slide Time: 13:14).

When we discussed about the load, there is a load dependent friction, there is other friction also, what is known as a spin losses, churning losses, that happens because of the spin velocity of the gear and they have this kind of profile, naturally dragging action will be there, and higher dragging friction will generate higher friction losses or higher spin losses. And many times, high speed gears, spin losses are far more or far greater than load dependent friction losses, but low speed larger side gear, this gear this load dependent on coefficient friction is generally higher compared to the spin losses.

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$$\mu_m = f\left(W_N, b, V_{pitch}, \eta, R_{composite}\right)$$

$\sqrt{R_{q1}^2 + R_{q2}^2}$ in microns		
Method	Before Running-in	After Running-in
Milling	2.3-4.6	1.2 – 2.3
Hobbing	1.2 – 2.3	0.9 – 1.7
Grinding	0.7 – 1.4	0.6 – 1.2
Lapping	0.6 – 1.1	0.4 – 0.9
Honing	0.3 – 0.6	0.2 – 0.4

Reference: W. J. Bartz, "Lubrication of Gearing".

Then, we discussed about, composite or surface roughness profile and we say this composite surface can be given as the R_{q1} square plus R_{q2} square and square root of this whole, this summation. R_{q1} is the surface roughness, may be say pinion R_{q2} is surface roughness of gear. We need to use this for calculating power composite or we say R composite can be given as a square root of R_{q1} square plus R_{q2} square.

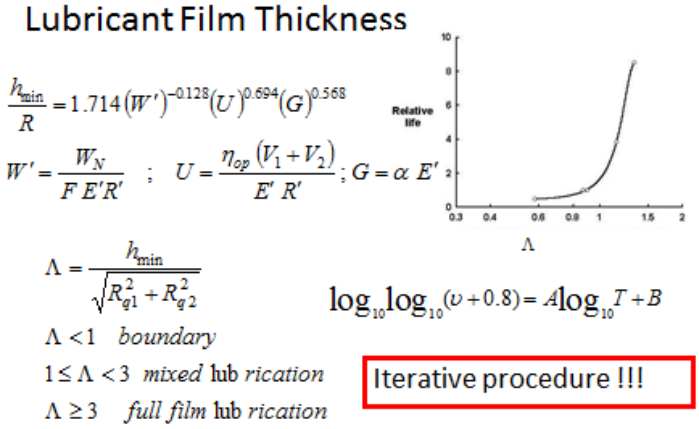
Now, it has been observed that when we manufacture the gears and start operating those gears, they have a different surface roughness compared to what we get after certain duration and that will be used the word running in time, this is shown over here, the milling operation, rubbing operation, grinding, lapping, honing. So, these milling and hopping generally give manufacturing processes, grinding, lapping and honing are super finishing operation to remove the asperities from the surface, but we are able to observe, here the surface roughness is the 2.3 to 4.6 micron. So, it is very high surface roughness and it cannot be used as it is, that means after milling, we require some sort of super finish operation.

However, if we are using this milling operation and may be, say for lower speed operation gears, we are using this. After running, may be say couple of hours or 100 of hours whichever the running in time, for the gears what we get is the lesser surface roughness is almost half of the surface roughness. So, what we use finally, when mathematical simulation time, we need to use a running in surface roughness, we cannot use start surface roughness, because that is not going to give reliable results and we cannot really operate also. When the surface roughness is generally high, we cannot use gears of the higher speed, we need to operate at

the lower speed and remove the oil whatever on the surfaces and change the oil and then come to this so that we can do this analysis.

So, surface roughness has importance, you say we need to take care of surface roughness after running in duration. And for detail, you can refer to this book W.J. Bartz, The Lubrication of Gearing as a complete book on lubrication of the gearing.

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Now, what we talk about the gear surface roughness is that, highly related to film thickness. When we are trying to decide which kind of lubrication, lubricant regime is going to work going to work on that case. So, this is an equation elastohydrodynamic equation, h min is the minimum separation between the gear pair which generally happens in some micron level and this is radius of curvature or the interface, it gives in a load parameter, velocity parameter, material parameter and load parameter which we have earlier discussed when we were discussing about the EHL elasto hydrodynamic lubrication mechanism. This depends on the load, depends on the phase width, here is the slight change there when we were discussing earlier, we used to, we were using the symbol F for the force, in case of the gears we are using the symbol F for the phase width, or we say that depth of the gear, width of the gear, E prime is a effective young's modulus, R prime is effective radius. We assume that their two surfaces and both are having some radius of curvature.

So, we need to find out effective radius of curvature, coming to the velocity parameter we say that it is depending on the viscosity, it depending on the velocity, summation of velocities, again it is also depending on E prime and R prime. Finally, this is the material parameter, it depends on pressure viscosity coefficient and E prime effective young's modulus.

So, we need to account these factors, what is the sensitivity, what we are able to see, they have cylinder, they have been simulated when they drive this kind of relation and of course, we can say that when we refer different books, different literature, everywhere this relation is slightly different, may be somewhere you will find this in a 0.073 or multiplication factor will be 2.65, this velocity will be in a power 0.67.

So, there will be delta variation depending on the experiments performed and this is the profit equation, we are not solving partial differential equation to give this kind of relation, this is more like profit equations and wherever the results are matching, they give coefficients according to those experiments.

As there will be always slight variation form one experiment to other experiments, and what I have observed, overall results are not going to be drastically changed, there will be delta variation, but that is within permissible limit. So, once we know these factors, we can find out what is the film thickness.

Now, once we know the film thickness, we can find out its specific film thickness which is going to decide with a gear under lubrication, mix lubrication or elastohydrodynamic lubrication. I am leaving the possibility of full film lubrication, there gears will not operate from full film lubrication, at most they can be operated in elastohydrodynamic lubrication, but we have a this kind of a division, we say that when lesser than boundary, lesser than 1, we need to say that this is the boundary lubrication, there will more and more asperities in contact and may be having some productive layer, lubrication layer of heard of nano meter to the micron may be say lesser than micron 1, 2, 3 is a mixed lubrication where the boundary lubrication is also existing under some external full film lubrication is also extending.

And this lambda greater than 3, we have seen the full film lubrication, but it is a elastohydrodynamic lubrication plus full film lubrication or we can say that elastohydrodynamic lubrication can be treated as a full film lubrication, because there is no contact. Generally, on the rolling surface near pitch circle diameter, this kind of a pitch surface is possible when there is a high pressure and is deforming the surface and is the lesser surface roughness.

So, this regime depends on both first is h_{min}, there is the minimum film thickness minimum separation and depends on the surface roughness profile and what we have seen in previous

slides that, if manufacturing process is not very good, what we are going to get high surface roughness and surface roughness may be the 3 micron or 2.3 micron or 3 microns.

Even after running in time, 2.3 micron and this is also 2.3 micron that means we need up keep the film thickness more than 5 micron between the gears. And if the film thickness is more than 5 micron, there is a possibility of lesser positive drive and that will also, again in those situations will be losing the efficiency from the gear, we want to operate gears as close as possible, but we want to avoid the wear also, it is more like a trade-off.

We want efficiency, positive drive, we also want no wear and that is why we need to have some sort of trade off. And it has been observed that as this lambda, capital lambda is increasing, when it is reaching to the 1, there is a sudden change in the relative life. If I compare when the lambda is 1.5 or with the lambda is 1, we found there is a substantial change or in another word boundary lubrication cases, generally gear is not going to give us very good life, reliable life, and that is the reason why we use the lubricant additives in gear oil. Whatever oil we use, we need to use some sort of E p additives, anti-wear additives, in those gears, if you are not able to use those gear additives, naturally performance will be lesser; it will be towards the boundary lubrication.

And gear life will be much lesser or relative gear life will be much lesser, but if there is jump from 1 to 1.5, we are able to see there is a jump, substantial jump, significant jump in relative line. So, that is important for us to find out what is the surface roughness, find out what is the film thickness, we see here film thickness itself is depending on number of parameters, if we increase the viscosity, film thickness is going to increase.

If we decrease the load, film thickness is going to increase, but sensitivity is different whether here sensitivity is roughly 0.7, while here the sensitivity is 0.13. So, there is a significant difference of 5 times difference in power. So, we need to see whether what is more suitable, increase in the speed or increase or decrease in the load whichever is more beneficial, we should go ahead with that, but again the trade-off will come whenever we are increasing the velocity.

Naturally, there will be more churning losses, there are more churning losses, there will be power loss, and we need to have some trade off in that. And whatever we have discussed in this, these are the fixed values so that in this case, we are using the viscosity, we are using the value of alpha, they are constant temperature values and we know alpha may not be that high

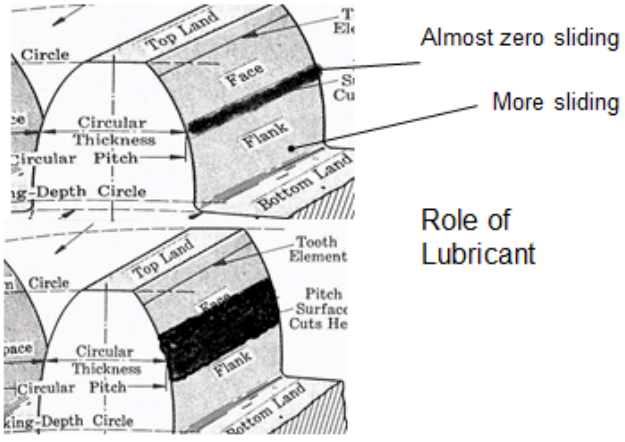
temperature sensitive, but viscosity will remain high temperature sensitive or sensitivity of viscosity is very high compared to alpha. As the temperature increases, this eta is going to change, and that can be given by number of relations, can be predicted, we discussed about two common literature, two common relations; one is Walter's relation other is vogel's relation, we know the vogel's relation required three constants while Walter's relation which is given over here requires only two constants.

Again, when we see the Walter's equation from a different book, sometime we get this constant as 0.6, 0.7 and 0.8, but we know very well whenever we are discussing about the gear oil, this viscosity, kinematic viscosity represented as centistokes, there will be in numbers, may be say 30, 40, 50, 60, 70, 80, 90. So, when we are talking about the 80 plus 0.8 is not going to make much difference when you are taking the longer scale.

Of course, this is the longer scale, means we know the viscosity of the two different temperatures 40 and 100 degree centigrade, we can find out the value of a and value b and once we find this, this kind of relation can be used iteratively to find out what will be operating viscosity.

Question comes, how to iterate this, we do not know the temperature, we have not discussed anything about the temperature, we are saying mu depends on regime; regime depends on the film thickness, and surface roughness. Now, film thickness depends on viscosity, viscosity is depending on the temperature. So, number of equations need to be solved together is highly complex, I mean good computer program will take may be say 5 to 6 hours minimum to give one result.

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$$Wear\ Volume = \frac{k_1 \times Load \times Sliding_velocity}{3 \times Hardness}$$

So, that is why many times what we say that we will use some sort of physical modeling or understanding based modeling, we know what is happening, we try to model on that. So, these are the couple of diagrams over here, we say that when we try to do modeling, we know there will be serial velocity here.

The coefficient of friction turn out to be 0 over here, there is almost 0 sliding, but here there is more sliding, we say gear tooth can be divided at the pitch level, above pitch, below pitch, this flank and face, generally the flank will be having higher wear rate compared to the faces.

That is why we say that, this portion is more subjected to fitting, is more subjected to the value compared to this portion, and we need to give very good thickness or thick over here. Similarly, when we talk about the sliding, here the sliding is much wider compared to over here and that is going to give the friction surface and this is what we say is more critical to analyze, and here importance comes from the role of the lubricant or we say lubricant gets importance over here.

If we use lubricant, first thing even in the rolling action, it is going to separate, it is going to generate elastohydrodynamic lubrication mechanism, it is going to separate two surfaces. So, lubricant is important over here. In addition, when there is a sliding, naturally lubrication is important, sliding is going to join in a higher friction and to reduce that friction, we require lubricant.

But problem is that, liquid lubricant itself gets sheared. So, it will give some sort of coefficient of friction, some friction, but when we compare solid to solid, sliding friction with the liquid, solid sliding friction, you find substantial change, that is why we prefer liquid shearing compared to the solid shearing. And here, it is also mentioned whatever you do as we are not going ahead with full film lubrication and even elastohydrodynamic lubrication happens only near to this surface, this circle, that is why there will be some wear and that wear can give sliding velocity, can be given by load, normal load, given by the material and hardness, material constant and hardness.

So, it is heavily dependent and now we choose a bad material having lesser stiffness, naturally the wear out will be more. If we apply more load, wear out will be more; if I apply more sliding, wear out will be more. So, many times we say that we can compare this as stuffed teeth, standard teeth and high contact ratio gears.

We say that this profile is extending beyond certain limit, that is going to give the high contact ratio, but along with the high contact ratio, it is going to every high sliding, high contact ratio is required from force point of view, higher the ratio lesser will be the force, obviously that lesser will be stresses.

But we have the contact ratio; there will be more sliding; more sliding, more friction. So, again when we do the friction analysis, we find there will be change and if there is wear of the gear, even though we keep a high contact ratio initially it is going to decrease with time. It is not going to be constant, this contact ratio depends on the time duration, there is more wear surfaces are getting damaged and then contact ratio will be decreased.

So, in other words we say that gear will never have infinite life; it will have only finite life depends on 20000 hours, of 30000 hours, 40000 hours. If you require more and more life, we need to require more and more wear volume even though whatever you do the good design initially, but when talk about the tribology, we need to reduce this wear rate, here wear rate can be reduced if you are able to reduce the sliding velocity, we are able to increase the hardness, we are able to reduce the load. So, one way or another way we need to reduce this k_1.

We know very well, k_1 is heavily dependent on the liquid lubricant, more and more lubrication this k_1 should be lesser and lesser. But larger and larger liquid lubricant is going to produce more and more churning losses. So, again the trade-off is required, higher friction may lead higher temperature, that may lead more volume of the wear.

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Procedure

- Determine effective values of velocity, load, modulus of elasticity and curvature.
- Determine/estimate effective temperature
- Determine viscosity and pressure viscosity coefficient as a function of temperature
- Calculate film thickness
- Compare film thickness roughness parameter and decide regime.

So, this is combination and that is why we required some sort of procedure, we say determine effective values of velocity, that is pitch circle diameter, you find out, you find out omega

and then you find out the value over there, then you find if you are able to calculate load and normal load, then effective modulus of elasticity, and effective curvature, effective radius.

So, that is the first step, when you start we need to generally have a data available to us, velocity can be determined based on the module number of teeth and that is which is going to give the diameter and omega and then load can be figured out, can be calculated if we know the power transmission or we know the torque applied on the gear.

That is going to give results of the load, then modulus elasticity or effective modulus elasticity, that require two material property, we say poison ratio and young's modulus and that is going to give the effective modulus of elasticity, then comes curvature. Here, we are taking the curvature, this curvature is different, this is the curvature, we are not talking about the pitch circle radius, we are talking about the convex shape of the gear and that curvature is important. We need to find out using pitch circle diameter and pressure angle, the pitch circle, and the curvature radius can be calculated using pitch circle radius and sine of the phi or sine of the pressure angle.

The multiple product is going to give us curvature, then next one is to determine and estimate effective temperature, we are not introduced many temperature relation, we need to balance or we say go for the thermal balance, thermal equilibrium to find out what will be the effective temperature, then once we know the temperature, we should determine the viscosity.

As we know, viscosity is going to get affected with the temperature, higher the temperature, lesser will be the viscosity and if possible, calculate pressure viscosity co efficient as a function of temperature, this also may reduce, then calculate the film thickness. Once we know the film thickness, compare it with a roughness and decide what will be the regime; once we know the regime, we can find the coefficient of friction.

So, coefficient of friction may be on a, if it is a mixed lubrication it may turn out to be lesser than 0.1, if it is a module lubrication turn out to be 0.1 to 0.3. So, depends on the regime and number of parameters, we can decide what will be the coefficient of friction. Again, we have given a bigger expression a big dependence, but those are indirectly depending on the coefficient of friction and indirectly affecting the coefficient of friction, what we say that we require so many calculations, that is one way.

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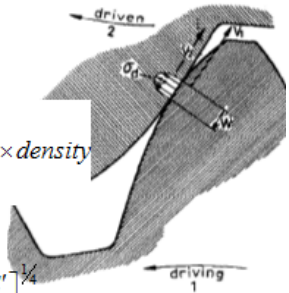
$$Rate\ of\ heat\ generation\ per\ unit\ area = \mu \sigma_d |V_1 - V_2|$$

$$T_f = 0.873\,\mu \frac{|V_1 - V_2|\sigma_d \sqrt{w}}{\sqrt{\beta_1 V_1} + \sqrt{\beta_2 V_2}}$$

$$Coeff\ of\ thermal contact,$$

$$\beta = thermal conduction \times specific heat \times density$$

$$\beta = \lambda C \rho$$



$$T_f = 0.62\,\mu \frac{|V_1 - V_2|}{\sqrt{\beta_1 V_1} + \sqrt{\beta_2 V_2}} \left[\frac{W_N}{F}\right]^{3/4} \left[\frac{E'}{R'}\right]^{-1/4}$$

$$\frac{1}{E'} = \frac{1}{2}\left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right]; \frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$$

But we can go ahead with other way, we can find out the effective temperature or maximum temperature, or the flash temperature of the gear interface using this relation, say what is this relation, we say his relation is been derived from rate of heat generation.

How to give the heat rate of heat generation, there is a mu which we are trying to figure out, this is sigma d that is the stresses which are causing the deformation. And this is the relative velocity at any time, what will be the velocity of the pinion velocity of the gear and then we are going to take the difference, naturally this velocity will be 0, this difference in velocity will be 0 at the pitch circle.

That is why we are not saying that, this T_f will come at any time at the pitch circle; it will be away from the pitch circle. The coefficient of friction also will be different; the pitch circle will be 0. Now, coming to the sigma d which is compressive stresses generated in the gear profile, we can see these are the convex shape and we can find out the radius of curvature of this surface. Once we know that this is the radius of curvature, we can find out what will be the effective radius of curvature that is R_I.

Similarly, if we know the material of this surface, material of gear and pinion, then we can find out young's modulus which requires the poison ratio of the gear material 1 and gear material 2. Once we have young's modulus, effective young's modulus, we have effective radius of curvature, we can go ahead with finding what will be sigma d,

because we know sigma d is a strong function of material properties and radius of curvature, which have been discussed in our the basic model when we discussed about the elastic deformation. Now, this is sigma d and we say that will be parabolic distribution, pressure will

be maximum in the centre and lesser away, this continuous variation in velocity will happen at every point will have a maximum pressure and may be say after 0.1 degree, 0.2 degree, it will have a lesser pressure.

Once we know, we calculate sigma d and substitute in this, what we are going to get this equation in terms of normal load and phase width; larger the face width. Lesser will be the maximum temperature. And in addition, what has been mentioned over here is, V_1 β_1 and β_2.

That β_1 and β_2 can be defined as coefficients of thermal contact, for the surface 1, and for surface 2, respectively. What is this thermal contact coefficient? We say that it depends on the conductivity, if I know the thermal conductivity material, that is going to give, can be represented as the lambda, is a small lambda and we know it need to have some sort of dissipation and the specific heat which can be represented as specific heat into density, we see as a specific heat and row as a density.

This is going to give us a beta value; larger the beta value, lesser will be the temperature or we say that if material conductivity is high, it can dissipate the heat faster and faster. Then naturally, there will not be accumulation of the temperature and naturally thermal equilibrium will be faster and we will get a lesser temperature, overall lesser temperature. So, this maximum temperature is depending on velocity or relative velocity. So, larger relative velocity, larger will be temperature and then it is also depending on sigma d which is also given in the function of load phase width, young's modulus and radius of curvature.

And this w, small w is this patch contact length, which in our earlier relation, in earlier lectures, what we represented this as a b or 2 b 2 into b, this is contact patch length, was represented if you refer earlier lecture we will get a similar expression what we are doing here, that is elastic effective radius of curvature, effective young's modulus, substitute we are going to get same relation. So, after substituting all, what we are getting, this is some multiplication factor into coefficient of friction, if we are able to find the boundary lubrication, mix lubrication and based on that lubrication, we can find out what will be the temperature.

Naturally, if the co efficient of friction is 0.1 and if I compare with 0.05, then co efficient this temperature will be reduced to half, reduction of temperature into half is significant. Usually, we should point out, there should be more and more additives to at least maintain boundary

lubrication and it should be some carrier fluid which can carry these boundary additives and try to keep some separation that will give lesser coefficient friction.

Then comes the relative velocity, and this is given in the expression β_1 into V_1 β_2 into V_2. Now, think over extreme cases, naturally when we are talking about the relative velocity, they will have negative velocity as well as positive velocity, what we are going to talk about extreme cases, assume that V_2 is not there, V_2 is 0, when V_2 is 0 what we are going to get maximum value of this, and V_1 will turn out to be overall, if I compare V from 0 to some positive value, this will give me maximum value of V_1 minus V_2 will not be V_1, if V_2 is positive.

That is the situation, this is V and here also this will turn out to be 0, this will be square root of β_1 into V and this overall expression can be modified, if we want to find out the extreme cases in trip conditions.

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Z	f_z
17	0.813
20	0.64
24	0.511
30	0.4
50	0.24
100	0.124

That is going to turn out to be something like this, in this case, V_1 divided by β_1 β_2 has been replaced and we know V_1 can be represented as omega into radius, that radius at any contact point, we are not talking about the pitch circle radius, contact point, that can be given in terms of number of teeth module. That can be given in those terms, because we know for the different gears are generally standardized, when the gears are standardized, naturally they will have some module, and some gear number of teeth and based on those, if this number r can be figured out can be calculated.

So, overall expression will turn out to be this omega, rotational speed divide by beta, load divided by phase width, do not get confused many times we get F as the force, but here we

are giving F as the phase width, it is in m, it is not in Newton while W is in Newton. Now, here m is a module, Z is the number of teeth and we are talking about this E prime as effective young's modules.

So, if you want to keep low value of teeth, what is required, we require a lesser speed, larger beta, larger phase width, lesser load, lesser module, that is very good thing, lesser module; in this case, lesser number of teeth and lesser effective young's modulus while we know that increase in number of teeth is going to reduce the load.

So, they are having some sort of dependence, when this dependence comes whatever the final comes, we need to take that into account. Now, here f z is depending on the number of teeth, that is the couple of numbers are shown over in this table, we say Z is 17, this value is 0.1, 0.813. So, here the number of teeth, increasing number of teeth is going to decrease the f z value, that is why I say and these proportionally too, here we are increasing the number of teeth, that is going to increase the temperature.

Increasing Z is going to increase the T f value, but increase in Z is going to decrease f z. So, this is more beneficial compared to this, it is only having the power of 4 or 0.25 while this in case power is 1. So, we need to compare it and wherever the benefit comes, we need to take a decision based on that, what will be the number of teeth can be decide so, based on the temperature calculation. We can find out the beta with thermal properties of the materials, we can find out omega from the rotational speed, whatever is been given and if we can find out the temperature individually. Temperature individually for the gear and individually for the pinion generally it will be more for the pinion in this case should go ahead with the pinion surface in this case.

And that this coefficient of friction depends heavily on surface roughness as well as the film thickness, film thickness itself will depend on number of materials wears there. So, we need to, because of clear design we talk about the gear design, it will not be simple like this, it will be based on combinations, number of teeth, young's modulus, effective curvature, load is P parameter. So, there is no one way rule that we need to have only these parameters.

It is always a trade-off, it is always a combination, it is overall package gives a lesser coefficient of friction and higher coefficient of friction, and we know the temperature is high, viscosity is going to decrease. If viscosity decreases, it is going to decrease the load carrying capacity. So, it is basically iterative scheme, it is iterative scheme that is why, we try to

utilize this kind of analytical expression, if we are requiring detailed calculation, it will take many iterations.

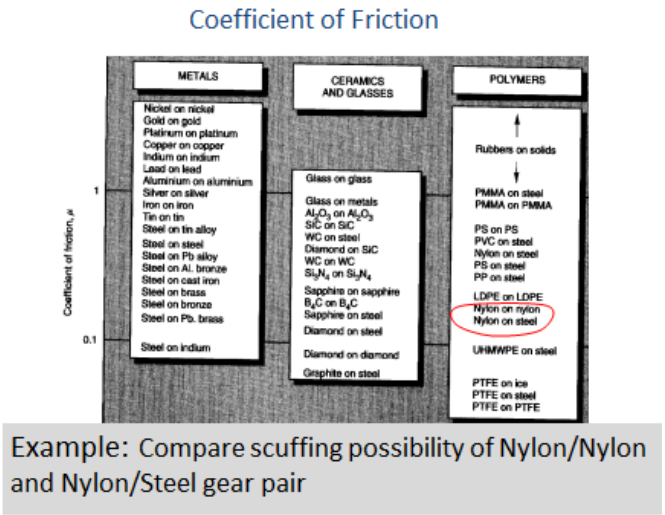
So, if we are designing initial level gears, we must always recommend this kind of equation find out the T_f find out the viscosity; again find out what will be the load carrying capacity, what will be the h minimum, because h minimum itself will be going to depend on the viscosity.

And when that is depending on viscosity, h minimum is going to affect mu and μ is going to affect the T_f and μ is going affect the T. So, that is always a perfect combination or we say we are going to get some temperature which is heavily dependent on itself, way.

So, this is what we say about pressure dependent coefficient of friction. We can calculate churning effect which is generally high, if viscosity is high churning effect will be higher side and that it will be required whenever there is speed of operation is very high. But generally, we say generally gears are recommended as speed reduction device, they are reduced, used to reduce the speed and increase the torque.

When we increase the torque, so in those particular applications, I feel the load is dependent, power loss is more dominant compared to speed dependent module, speed defined on churning effect may turn out to be lesser than 50 percent when we talk about the real application of gears. Generally, we do not use the gears for high speed application, but if it is required, then we should calculate the churning effects, otherwise using only the pressure load dependent friction losses will be sufficient for us.

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Example: Compare scuffing possibility of Nylon/Nylon and Nylon/Steel gear pair

So, we will see how to calculate, we will just take one example to find out, how to utilize this relation. See, there we know the co efficient of friction at different regimes, if it is coefficient friction in dry case and I mentioned about one of the examples, that polymers can be used as gear material. When we use polymer as the gear material, we should check, what is the validity of those gear material, that is why we are taking possibility of scuffing of high temperature, mean temperature is very high there is a possibility of plastic deformation, plastic flow and two materials will get welded and that is known as scuffing.

This scuffing is one of the failures, we generally do testing of this scuffing on various gear machines, but if there is a proper design, proper design of the gear, proper design of the lubrication mechanism, there will be very low probability of scuffing, but it is possible when we use some sort of polymer gear material, we need to find out is there any possibility of scuffing.

So, example says, compare scuffing possibilities on nylon, versus nylon steel gear pair, here again we are using two nylon gear, but here we are using one, a steel gear and one nylon gear. We are not talking about the steel, we are talking about the nylon steel and when we compare the co efficient of friction, what we find, nylon and nylon coefficient of friction is slightly more than nylon on the steel, slightly more. So, we can start with a dry lubrication, we say there is no liquid lubrication and mu can be taken as it is, it is a dry lubrication mechanism.

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- **Given data:** Z=17, m = 3 mm, b =30mm, ω=150 rad/s, Power = 850 W.

$$T_f = 0.62\,\mu \sqrt{\frac{\omega}{\beta}} \left[\frac{W}{F}\right]^{3/4} [m\,Z\,E']^{1/4}\, f_z$$

Coeff of thermal contact,

$\beta = \lambda\, C\rho$

	N-N	N-S
β	0.417e6	180.e6
μ	0.2	0.2
E'	20e8	40e8
	161°C	19.5°C

So, we can implement that and try to find out, maybe we require some data for calculation. So, for time being, we have taken the pinion gear teeth as a 17, module as the 3 mm, that is going to give us a 51 mm pitch circle diameter.

And if you want the radius, we can divide by 2, that will be pitch circle radius, phase width in this case, what I have written b is of a 30 mm, and omega is a 150 radiant to power second, while power which is been transmitted from nylon gears is slightly lower side is 850 watts. So, based on the power, we do can find out power and omega and pitch circle radius, we can find out what will be the W and of course, I am assuming there the pressure angle will be 20 degree, without pressure angle will be very difficult for us to get the results, then this is phase width and this is by mistake it is written as b here, but it is b f.

So, this b is 30 mm or b is f and f is 30 mm, the module has been defined number of teeth are defined and we can find out young's modulus, effective young's modulus based on the material pair, we know the nylon material pair, we know the other pair is nylon steel and young's modulus on the materials are known to us, poison ratios are known to us, we can substitute those values, we can find out and this table is going to give showing the results.

That is, when we talk about the nylon, nylon effective young's modulus is roughly 2000 mega Pascal or we say 2 gigapascal, while when we talk about nylon versus steel, we are forgetting its roughly 4 gigapascal, otherwise young's modulus for this steel is very high, it is almost 200 plus gigapascal. But talking about the pair, naturally lower value will be dominating, that is going to decide what will be the final value and that is showing the 4 gigapascal.

So, once we know E prime, we know Z here, we know m here, we know F, we can find out W, this omega has been given to us, and coefficient of friction from previous table we can figure out as 0.2 nearby 0.2, even though we know that nylon versus steel will give slightly lesser coefficient of friction compared to nylon versus nylon.

But we can take same value; almost same value is 0.2, 0.2 to find out what is the relation or what the overall temperature is, is there any possibility of scuffing, is there a possibility of high temperature. So, once we use this parameter, of course there is a beta parameter which can be figured out from the material combination and here we are able to see that.

Beta for nylon versus nylon is roughly 0.417 10 is to 6, while in case of nylon versus steel, because of thermal conductivity. Because of high thermal conductivity of steel, this is substantially high value, compared to 0.147 is at 180 almost 360, almost of the 360 times, very high value, high compressor and that is in the denominator.

So, higher value of beta will be preferred to keep temperature lower side, and that is what is going to happen, you can see here for nylon versus nylon, the temperature is the 161, very high temperatures for the nylon to bear it, Of course, within number of fillers, the number of fillers can be increased, but still we cannot say there will be elastic deformation, there will be perfect plastic deformation, they are when temperatures are this high, naturally contact will be subjected to slightly plastic deformation, it will not be only the elastic deformation.

While coming to the steel versus nylon, this temperature is only 19.5 degree, very low value compared to 161, it is not only the one eighth of that or 12.5 percent of that, but with increase in temperature, sensitivity changes drastically. So, this combination will be preferred or we say that whenever we choose the gears, may be one polymer gear, but other solid material gear or which has a very good hardness, which can have, which has also the high thermal conductivity. If the gear has hardness and thermal conductivity, surely we can use this kind of pair, one solid material pair, one solid material gear of pinion and other as a steel or some material which has high thermal conductivity, that will be preferable compared to this kind of thing.

But we know very well, for the number of application in home appliance, this nylon gears are used, given the reason being their duty is lower side, they do not transmit power continuously, and it is generally recommended for the mixture. We operate the mixture, we say we do not operate for 10 minutes, because it we continuously operate for 10 minutes, the temperature will continuously increase.

And we may find that, gear versus gear is going to get engaged or they will be folded, it cannot be separated, it cannot be rotated after that. And that is the reason we want to keep operation for some time, not beyond certain level, because they will be thermal instability, there will be more flow and there will be more flow cold welding, and this is an important aspect whenever we choose a gear material, we need to think about good thermal conductivity, because we know the contact point there will be high temperature.

There is a possibility of high temperature and there is a metal to metal contact and in this whole example, we took coefficient of friction is equal to 0.2, but if we provide some sort of lesser coefficient of friction, that is going to affect the results. If I decrease this coefficient of friction, may be say ten times, naturally this temperature is going to reduce 10 times. But in that, in those situations, nylon selecting nylon will not be any use, it can be removed or we can choose some alternative material. So, we will continue our gear topic in our next lecture.

Next lecture, we will be considering what are the possibilities of gear failure, we have explored possibility of gear failure in this case, particularly scuffing failure which happens at high temperature of the lubricant or very high coefficient, because of the high coefficient of friction and lesser thermal conductivity, but there are other possibilities, or possibilities of failure of gears, we will be exploring in our next lecture, thank you.