Tribology

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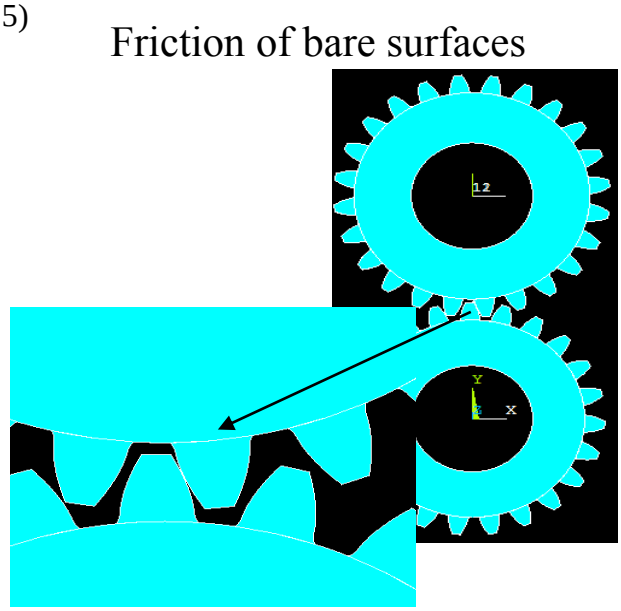
Module No. # 06

Lecture No. # 36
Friction and Lubrication of Gears

Hello and welcome to 36 the lecture of video course on tribology, topic of the present lecture is Friction and Lubrication of Gears. In our previous lecture we started gears, we defined gears, defined profile of the gears and mentioned there will be some sliding, other than pitch points, and that sliding causes a friction. In addition, as we use lubricant to reduce friction, but there will be another disadvantage that will cause churning effect.

Only one or two gear teeth will be in match, remaining gear teeth will be without machine, so they start splashing a lubricant and there will be some loses, frictional loses, and energy loses, because of that. We are going to discuss this topic in a present lecture.

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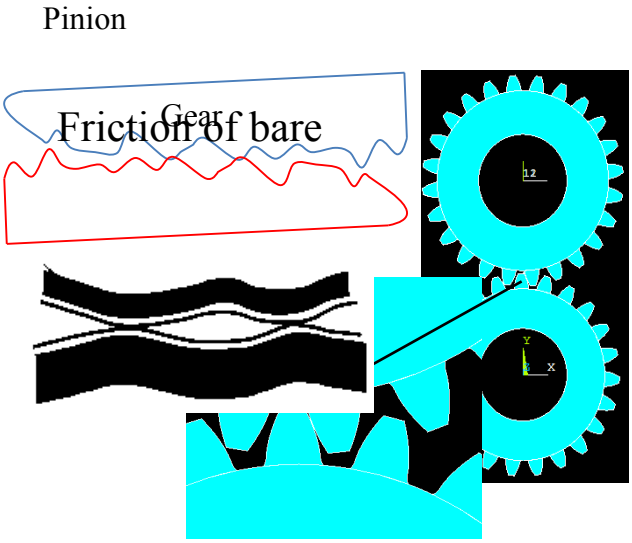


We can start thinking about the bare surfaces without any lubricant. When we are discussing about the friction at the different tribo-spheres, this was the first what we call as a dry friction. And to illustrate it, to demonstrate it, we have shown in this gear pair in the match,

and we are able to see, there is a matching at this point, if I zoom it we are able to see the match, and we are able to see some sort of elastic deformation on the surface.

If surfaces are without any lubricant and they need to rotate, naturally there will be high friction, coefficient of friction may even reach 0.6, 0.7, if surface finishes are not very good, surfaces are not harder they deflect easily then, that will not be desirable, because of gears are used for the power transmission, not for power losing effect. We want to transmit the power as much as possible, efficiency should be 100 percent and if we take care properly we can reach to 99 percent, 98 percent; reaching beyond that will be just impossible unless we change the technology, from mechanical point of view that will be pretty difficult.

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Now, this can be demonstrated why there is a friction. You are able to see there are some asperities, whenever there is a mechanical contact, under load condition there is a possibility of deformation of the surface and because of the asperities, there will be inter locking, and this inter locking may be based on just abrasion or may be based on adhesion of the cool welding.

Particularly in dry surfaces, cool welding is predominantly phenomenal, coefficient of friction majorly occurs, because of the adhesion, because of the cool weld and the asperities; and we need to reduce as low as possible, that is how we say this figure shows this asperity to penetrate on another surface, it will be difficult to remove the pinion over the gear.

However, we know very well that, in normal atmosphere condition there will be always one contaminated layer on the metallic surfaces, while we are discussing over here, particularly

for the metallic surfaces and mostly gears are manufactured having metal surface, even though we know very well that polymers have started being used to manufacture gears, but for the low load applications.

Even in this case or those cases also there will be frictional loses, what we are talking here only for metallic surfaces and we say the metals, if kept in environment, it will absorb few layers may be to the nanometer level thick layer, but there will be some kind of layers on a surface and that is shown with this white colour layer on the surfaces, we are showing that, the both the surfaces may be the pinion surface and gear surface.

Both will have some sort of contamination layer, without any lubricant these layers are generally having a lesser shear strength, that is why the relative sliding will be simpler compared to the parent material, compared to the bare materials, without any contamination layer on that.

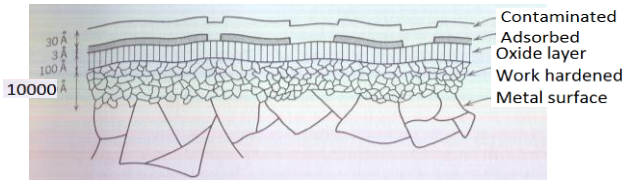
And this has been studied in our friction module lectures, may be that is in 3rd, 4th lecture in which was discussed in detail. If there is a bare surface and kept in environment, there will be some sort of oxidation and that oxidation itself will generate low shear strength, lubricant layer on a surface, but still that can be treated as dry lubrication.

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1-2 μm thin coating. Few microns



$$\mu = \frac{\tau_i}{2\sqrt{(\tau_y^2 - \tau_i^2)}} = \frac{0.5}{\sqrt{\left(\frac{\tau_y}{\tau_i}\right)^2 - 1}}$$



Now, as this is being shown that, there is a contamination layer on that, the question comes, can we really go ahead, can we design this lubricant layer on our own, using some sort of solid lubricant, we have studied solid lubricants, we know about the molybdenum disulphide, we know about the graphite surfaces, even we know the coatings, those are the solid lubricant

layers. It can be designed as per the requirement, while what we discuss in to the natural layer, natural layer we do not have much control, it will change with the environment, we are operating at the 30 degree centigrade, though there will be lubricant thickness on a surface.

Now, we start operating 70 degree centigrade there is a possibility that the layer get thicker or there is a possibility that the oxidation rate increases, but there is a possibility also that they will start softening and the because of the soften effect there will be more and more cold welding, their hardness will reduce. So, coefficient of friction will continuously change as the temperature is going to change similarly, in environment if there is more and more moisture, coefficient of friction will change, because of the thickness of the layer.

To avoid that kind of uncertainties, we can think about solid lubricants, we can coed these surfaces with solid lubricants, one of the very popular solid lubricants used for the solid, for the gears is a phosphate. So, that, we use a phosphate layers on solids as a solid surface, solid lubricant with a gear surface. Now, there is our relation that shows if thickness, if the shear strength of coating is lower, then coefficient of friction will be lesser. Naturally, when we are talking about the shear strength of the layer, it is a relative term, is the relative to parent material τ_y is a parent material or we say they are gear and pinion which they are made of the same material, then we can take any τ_y, but the gear pinion is softer material compared to the gear or the hardness is lesser or shear strength is lesser then τ_y will be the pinion material layer, we say the shear strength of pinion material.

Now, this tau y is interface layer whatever you say interface layer meet, if we are using solid lubricant naturally the shear strength of the solid lubricant will come, if this ratio is large we say may be τ_y is 99 times of τ i. In that case, it will be very huge number and coefficient of friction will decrease drastically, but if we say now τ_y is just 20 percent higher than τ_i naturally this factor will be 1.44 minus 1 so square root of this and that will factor still will be very high, the coefficient of friction will be high. So, depends what is this ratio, if τ_i is much lower than τ_y coefficient of friction will be much lesser, and we do think solid lubricant from that point of view.

And this is diagram that, shows how if we keep gear surface in environment there is a possibility of some contaminated layer on top of surface, the contaminant layer is a oxide layer some sort of a work hardening layer. Instead of relying on this layers which will change their thickness as per the environment we better design lubricant layer as per the requirement, we think about the phosphate layer, we can think about the graphite layer, we can think about

the molybdenum disulphide layer. And whatever is required, we can design accordingly most often these solid lubricants are used in a liquid lubricant.

We know very well, if there is a solid lubricant deposited on the surface what will happen, under rubbing condition that thickness layer will slowly reduce. It will not remain same or in other words, we are going to avoid the wear of gear material, but we cannot avoid the wear of solid material or solid lubricant material; if we are using Teflon, naturally Teflon powder form will come out or we say will be born out from the coating surface.

Similarly, if we are using the molybdenum disulphide and there has been condition naturally molybdenum disulphide will be simply slowly removed from the surface, same thing with the graphite sulphide also, so in gears particularly we cannot avoid the wear, we can only divert the wear from the gear surface or gear material to the solid lubricant material. We are not avoiding, we are not making it 0, where wear will be there, but just we are changing the wear from gear material, so that it can be diverted to the solid lubricant material.

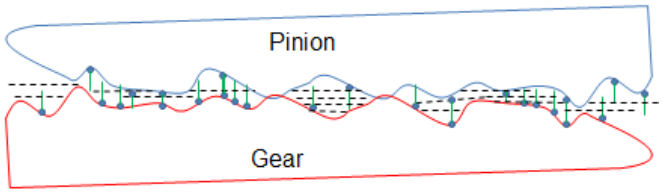
Now, if there is a liquid lubricant what will the liquid lubricant do. It can carry those powder along with it and it can be again a replenished or can be used again and again. So, there will be always some constant thickness layer depends on equilibrium, physical equilibrium, chemical equilibrium there will be layer and that is advantages.

Often we say 1 to 2 micron thin solid lubricant layer is sufficient, you do not require more than that, we know very well there will be some sort of wear out obviously lubricant layer and we do not want any time the lubricant layer turn out to be 0, so we generally keep few micron it can be 10 micron, 15 micron depends on the micron of the components and depends whether we are going to use liquid lubricant along with the solid lubricant on that. If you are not using any liquid lubricant naturally we need to keep slightly thicker layer, so that component can be refurnished after certain number of hours or certain number of operating hours.

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- Phosphate

$$\mu = \frac{\tau_i}{2\sqrt{(\tau_y^2 - \tau_i^2)}}$$



$$\tau_i = W_1 \tau_{mi} + W_2 \tau_{bli} + W_3 \tau_{EHli} + W_4 \tau_{FFli}$$

$$\tau_i = W'_2 \tau_{bli} + W'_3 \tau_{EHli} \quad \tau_i = \alpha \tau_{bli} + (1-\alpha)\tau_{EHli}$$

So, this is what a mixed lubricant was more like a dry lubrication with a solid lubricant, if I say more appropriate word solid lubricant also can be treated as a boundary lubricants. They are making a boundary layer. But, what we are talking when we use a liquid lubricant along with the solid lubricant, then situation will turn out to be mix lubrication what is the reason for that, here the load carrying capacity will be or sheared by liquid lubricant, solid lubricant, metal to metal contact and full film lubricants. So, all four are mean of the lubrication, all four mechanism of lubricant will be there or we can say this is mu coefficient of friction we are using the same relation, which we have shown in previous slide, but this tau I can be calculated as a weighted summation of different shear strength.

So, shear strength or metal interface shear strength or bond lubricant interface, shearing strength of elastohydrodynamic lubrication mechanism, shearing strength of fluid from lubrication. So, all lubrication mechanisms are present when we talk about the mix lubrication that is a name, we say that there will be solid lubrication and there will be liquid lubrication, so naturally it will be going to be a mixture of that and we talk about the solid lubricant, we know that there will be metal to metal interface, there is a possibility of boundary layer interface. When we talk about fluid from lubrication, we can talk about the elastohydrodynamic lubrication we talk about the full film lubrication with complete surfaces are separated by liquid lubricant and sufficient thick lubricant layer is there when there is no deformation, there is no elastic deformation of the surfaces.

So, there is a possibility and when we talk about the metal surface and we talk about the designing of those metallic surfaces particularly for the gears, naturally we will not tolerate this metal interface, we do not want any time gear material come in a contact with other gear

material, so this shear strength can be avoided. And of course, use a W_1, W_2, W_3, W_4 summation of this $W_1 + W_2 + W_3 + W_4$ will be equal to 1, we say that these we are using in a normalization form, we say that these are weighted, these are the weights, should be given along with individual shear strength. Discussing about we cannot tolerate metal contact, so we try to avoid as far as possible in all situations, so this parameter may not come when we talking about the frictional force for gear pair this will not come similarly, this will not also come we cannot tolerate the full film lubrication.

We cannot transfer the motion 100 percent without any loss of the velocity using the FFL that is why this will not work this is not going to work only two terms we

are going to work with bond lubrication and we are going to work with elastohydrodynamic lubrications or no word pressure or contact load will be immense in this situation that is going to cause a deformation of surface, deformation of tooth surface and we will be using some sort of boundary additives, it will not be the only liquid lubricant but, we need to use the boundary lubricants. So that, wear of the surface is minimum when it extends to the elastohydrodynamic lubrication. Even we say that there is a mechanism like that, they still the surface that comes in contact.

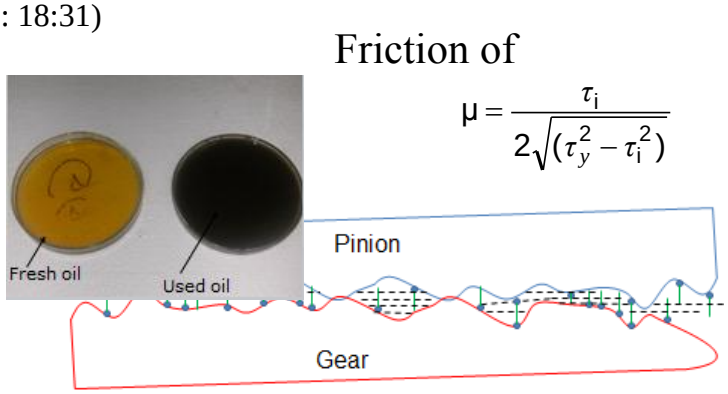
And to avoid that metal to metal surface we need to use boundary additives and now if I you say that τ_{mi} is not going to work τ_{FFi} is not going to work, naturally this will be 0, this will be 0 and we have only two mechanism that is a bonded lubrication mechanism and elastohydrodynamic lubrication mechanism. Now, this two mechanisms naturally their weights will be different now, initially may be this was 0.1, 0.2, 0.3 and this value has something like 0.1, 0.2, 0.3.

When we talk about the gear lubrication mechanism naturally this W_1 is 0, W_4 is 0 that is why we are not taking, we are not accounting tau m i and tau F l naturally this two values will differ. Now, there is a W prime 2, W prime 3 and this summation will be equal to 1 are in other way, so W prime 2 plus W prime 3 will be equal to 1.0, in other words if I say alpha is used as a weight factor, naturally other factor will be 1 minus alpha, so this is a proper shear in strength formula which can be directly substituted here and we can figure out what will be the relative coefficient of friction.

We are not talking about the absolute coefficient of friction, whenever we compare this is going to be good competitor, but if we talk about the absolute coefficient of friction it

depends on many parameters, many factors and we cannot really estimate mathematically what will be exactly coefficient of friction on the interface, we need to do extensive experiments for that. But, if we want to compare different material pair, if we want to compare different harnesses, if we want to compare different look lubricant, if we want to compare different profiles, this kind of relation can be used.

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$$\tau_i = W_1 \tau_{mi} + W_2 \tau_{bli} + W_3 \tau_{EHli} + W_4 \tau_{FFli}$$

$$\tau_i = W'_2 \tau_{bli} + W'_3 \tau_{EHli} \quad \tau_i = \alpha \tau_{bli} + (1-\alpha)\tau_{EHli}$$

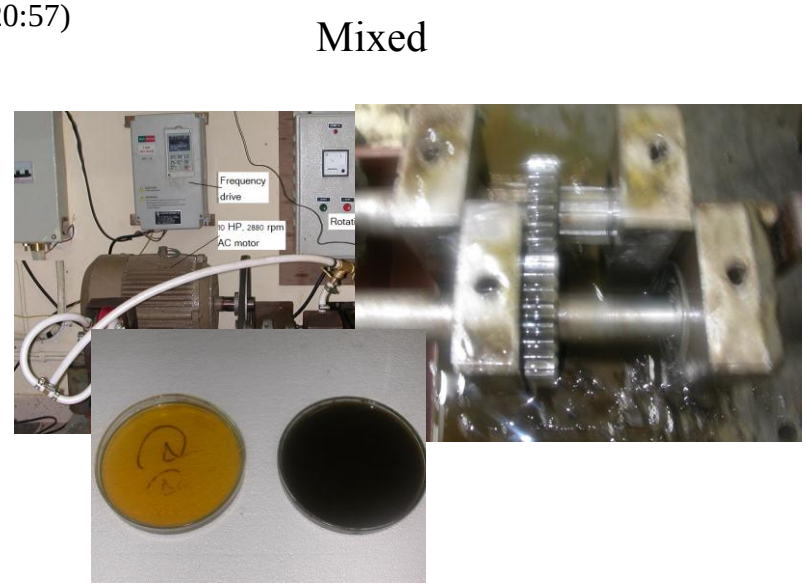
So, this is a relative parameter or we say this can be used as a competitor, but cannot be treated as an absolute value and this is what we are showing in figure where the image of a liquid lubricant, is shown in photograph. What we did experiments on gear pair in our lab and what we found, when we were using initially this kind of the fresh oil, yellow oil and good oil it shows that it is a liquid oil after doing a 100 hours operation, when we removed the oil from gear box we found this kind of a colour, this is completely black and we could not find separate particles, but this liquid is filled with all the wear particles.

Now, this is still surface damage happening and we can say that is a clear indication that whenever we operate gear pair, they are going to work with mix lubrication mechanism, they are not going to work with full film lubrication they are not only going to work with EHl.

So, this two lubrication mechanisms similar, may be last one can be rejected this can be considered but, there will be boundary lubrication and if there is no sufficient boundary additives available, then metal interface will occur, but now we are more careful about the gear pair. We know manufacturing cost on a higher side and they are very sophisticated components, they are very important components naturally will be using some sort of boundary additives, and whatever the boundary layer is made of will be with chemicals.

Lubricant layer is not on the solid lubricant then this kind of a lubricant, this kind of colour of the liquid lubricant will emerge that is obvious. So, if the chemical action is happening or chemical layer on the gear pair, naturally some metal is going to act along with the oxide are getting oxidized and when it is removed naturally it is going to make a colour like this.

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So, that is important to consider, what I was talking about what we had done in the lab, so this is a set up which is just demonstrating, what we did. This is a 10 hp motor this is a belt for drive and this is a gear box. Naturally gear box need to be loaded that is why there are connected with a spinal coupling and that is a dynamometer.

This dynamometer was made in house, we could apply up to 20 Newton meter torque but, completely controlled condition when we operated for the 100 hours what we got, when we opened the gear box what we are getting is completely filled with a black colour oil and even this particle you can see that the kind of the colour of the oil here. If we use a fresh oil it will not be this kind of thing, it is generally yellow colour oil but, here it is turning out to be slightly blackish and these are the gear teeth, this is also having some sort of the black oil on the surfaces of gear teeth and this is accumulation of some wear particle in this oil.

So, when we do experiments, 100 percent know that, it is not going to be a full film lubrication mechanism and we are using liquid lubricant, naturally there will be wear because of either direct removal of material or some oxidants of that matter.

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With involute profile of gears, only one contact

In the case of crossed axis rolling gears, velocity component in axial direction. Some sliding at rolling point !!

Mixed Friction

Type of Gear	Efficiency (%)
Spur	97-98
Bevel – Intersecting axes	97
Bevel – Offset axes	94-96
Cross helical	84
Worm	30- 97

Now, that clearly indicates when you talk about gear pair there will be wear of a surface, but this wear can be reduced when we talk about the power loss or when we talk about the friction loss, we can say the two surfaces they come in contact they need to have a lesser friction resistant, but there is a problem we require intimate counting between two surfaces, so that relative velocity is 0. I would say relative velocity is 0 on that is what we say the efficiency, we want complete velocity to be transferred from one surface to the other surface and that is basically with the friction mechanism, because we have used an invalid profile or the positive drive profile for that, but still that is a problem.

We want minimum friction and we want good friction also, so we come out with some sort of trade of, when you talk about the friction force and we know very well that friction force is going to work between the gear teeth, but we cannot term as it is reason being that there is velocity which does not remain constant or we say the sliding velocity does not remain constant, is sliding velocity will be 0 at only on the pitch point.

Away from the pitch point velocity is going to change and if of course, in some gear pair even there will be sliding at a pitch point, which we have studied in previous lecture and I am just repeating this slide with a few observation from the previous lecture, while we say that with invalid profile of the gears. Only one contact position experiences pure load, pure loading happening only at one point, now as contact moves towards or away from the pitch point sliding is going to occur or sliding is bound, so that sliding will continuously increase, if we move away from the pitch point.

Then another example was k1, we say in the case of crossed axis, rolling gears where the axis are not parallel, they are not intersecting, they are not coplanar and they are out of the plane, then surely there will be some component in the third plane or third direction also which is going to create additional sliding. That is why you see, in the case of the cross axis rolling gears, velocity companion in axial direction will occur and that is going to increase the sliding, over all sliding will increase in this situations, that is why when we try to compute different gear pair or we say that different gears, what we mean type of the gears & efficiency.

Simplest way to manufacture, easiest to analyze is spur gear, we say that shaft axis are parallel and length of a bearing, gear width is also parallel to the axis; efficiency is pretty high it is 97 to 98 percent. Of course, we are talking about the good condition, we are not talking that we are using, that we are not going to use any oil or something like that we are talking about a good conditions, all good conditions; this spur gear shows 97 to 98 percent efficiency in good conditions. Coming to the bevel side, in previous lecture we mentioned there is a spur gear works on cylinder as a cylinder, while bevel gear works as a cone on cone they role on each other.

So, bevel gear when axis are intersecting may be 90 degree, 60 degree, 70 degree depends on kind of geometry, they show again an ideal condition, good operating condition efficiency of the 97 percent. However, if we offset its gear axis, there is some gap, efficiency is slightly lower than 94 to 96 percent, we say that offset is very minute then efficiency will be high, offset on greater side then efficiency will be 94 percent.

Now, these are all parallel axis related options when talk about the cross helical that coplanar the surfaces are not there or the axis are not in one plane, then efficiency because you can see is reducing drastically. Helical gears otherwise are good non gears or they work with a lesser noise, much better performance, when we are talking about, crossing of that or we say that instead of line contact they have point contact then efficiency is dropping from may be say 100 percent to 84 percent that is a substantial change.

Well when we work with one gear, the torque ratio is almost 42 worm it is very high torque ratio, but sliding also will be very high in those situations, so efficiency goes down in this situations, when the sliding is very high naturally efficiency is just 30 percent, when we are able to control it properly relative low sliding velocity, the efficiency can be reached with 97

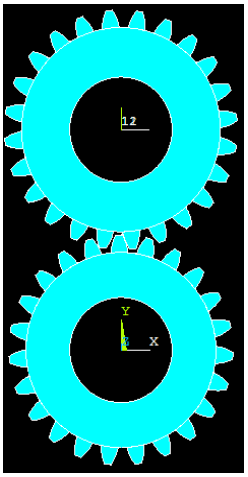
percent, but of course, these are the very exceptional cases generally efficiency of one gear is lesser than 90 percent.

So, this gives an idea that, these are having lesser efficiencies we say the efficiency having lesser than 100 percent, naturally there will be some sort of power loss and we can talk about the frictional loss and multiply with the velocity but, that is not in our case or will be very simple case. Here, the sliding velocity is going to change continuously as we move away from the pitch point and it always happened in one contact we say tooth contact, only one point will be at the pitch point all other points may be say there are 200 points, 300 points whatever the way we divide it, there will be different sliding speed and they were continuous point, there is change in the sliding velocity naturally we can have to talk in terms of power loss, we cannot talk just in terms of friction losses.

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Power Loss

As contact moves towards or away from pitch point, sliding occurs.



So, we say one of the major factor in power loss when we talk about the gear box naturally there will be shaft, there will be bearings, and all that when we just talk about the gear pair, only the contact is happening at the tooth naturally there will be power loss, however if we talk about the these surfaces, this tooth surfaces are not under load. So, power loss due to the load will happen over here only, while power loss without load will happen on number of other teeth, as there will be some sort of dragging action this is the solid metal and if it is dipped in the liquid lubricant naturally splashing will occur; and will try to splash lubricant out, you can see there is a discontinuity surface, if there is smooth surface then there will not be problem, but this surface, because of the massive requirement discontinuity.

Naturally, there will be more and more splashing loss, more and more I can say idle power loss, we are not talking about the load transmitted through this systems or we can be talking about the churning effect, it is going to churn like a lubricant or going to cause loss of the load. If viscosity of the liquid lubricant is on higher side naturally this churning noise will also be on the higher side, so we have two power loss we will talk about the teething power loss is an idle power loss, when the load is not there and there will be load power loss. And naturally when we talk about the load we talk about the elastohydrodynamic lubrication, we talk about the mix lubrication actually this component is going to play a major role.

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Constant or variable ??

Toothing Power Loss

$$P_{load} = F_r v_g$$

$$P_{load} = \mu W_N v_g$$

- Idle power loss

$$\mu_m = f\left(W_N, b, V_{pitch}, \eta, R_{composite}\right)$$

And this can be given as F_r frictional force into velocity into sliding velocity, to precise side we say frictional force should be presented in terms of a normal load, and that is a μ normal load is a μW_N, but the coefficient of friction is a μ over here, so it is a μW_N into v_g is going to change. Question comes, when mu be changing really or is going to be constant, will W_N change or will remain constant, so these two are important aspects μ cannot be constant.

We know loading points are going to change, applied load is going to be change, it may vary from 0 to the. Newton load as load is such high variation on the point of contact; naturally mu is going to vary continuously. Talking about W_N again the point of contact is continuously changing or line of contact is continuously changing, naturally this W_N will not remain fixed, is also going to change.

So, μ is going to change, W_N is going to change and v_g is also going to change naturally, we are talking about the power loss, which the average power loss and mean power loss and that is why we say that, these parameters are not constant they are going to vary continuously in magnitude, may be in direction also. And interesting thing is that μ, itself is a function of

number of parameters, that is why, is it in a μ_m this is a mean value, average value of coefficient of friction it is the function of itself, load is going to depend on the load itself, it is going to depend on the bearing or we say the gear length.

If the length is increasing, this loss will be high or a μ will be higher side, is going to depend on a pitch velocities or summation of pitch velocities, is going to depend on the liquid lubricant viscosity fluid viscosity and it is going to be a function of surface roughness a larger the surface roughness, larger the value of μ.

So, this is difficult to find out through the signs has been shown, if I take a constant value, if I change any of this parameter mu is going to change. Of course, sensitivity of the parameters is different, talking about this R the surface roughness; it is a composite surface roughness. Naturally, the gear frank surface roughness will be a constant, if a gear frank or a pinion as well as the gear frank of the gear or say surface roughness of both the surfaces will be counted, and generally it is a RMS value which is to be counted. So, we know μ of the μ even the mean value is going to be function of so many parameters or many times is difficult to prove through the signs.

But, when we do number of experiments we can find out using equation we substitute this parameters 3, 4 level parameters, do experiments and find out what will be the μ. So, it is going to be decided based on experiments or equation with some unknown constant which need to be determined through experiments.

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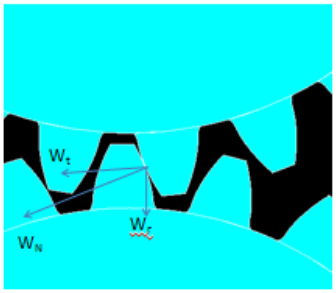
- Idle power

$$W_t = W_N \cos\phi$$

$$W_r = W_N \sin\phi$$

$$W_t = \frac{T}{r_p}$$

$$W_N = \frac{T}{\cos\phi\, r_p}$$



Contact ratio ???
1.4
2.0
High contact ratio

Now, W_N, question comes what is this W_N, see this is a gear pair now any surface perpendicular to that, that should give away normal load something like this, this is a W_N question comes how to find out from where this figure will turn out, what is mechanism? we have some sort of pressure angle and we know pressure angle if I account a pressure angle I

will be getting two components, we say W_t components which is going to drive the gear pair, and W_r that is a radial load is going to push gear away from other gear or is going to be analyzing bearing, so there will be radial load on the bearing.

So, how to find out this W_t and W_r, that is another question, when we generally know what is a, what power to be transmitted when we know power to be transmitted and that too in a rotational direction; so we can figure out this however, before that we need to find out relation between W_N, W_t and W_r and the see that W_t will be W_N into cos phi. Actually W_N is going to be greater than W_t and W_r or we can say we can take square root of this is vector summation of W_t and W_r we can find out W_n, so this W_n will be maximum load, and W_r which is there need to be minimized, so that bearing is less loaded or subjected to less load.

Now, we talk how to figure out W_t or we say that any of this W once I know that any of w and I, if I know the phi value of course, this phi again will change the pitch circle diameter generally is known, but away from the pitch circle diameter is generally not known we need to find out from the geometry.

Now, this W_t can be given in terms of torque by pitch circle, we say the radius, pitch circle radius for pinion will be r_p similarly, for gear it will be r_g and generally r_g is a lesser, r_g is a more than r_p that is why we are using this relation, we want on more on conservative side and that is why we are taking higher value of W_t. So, W_t is a torque, torque will remain same in the both the cases or and not remain the same, power will remain same, the torque will remain the same, so torque whatever torque on the pinion and divide by r_p that is going to give us a relation. Now, once we know this we can figure what will be the W_N and that is turning out to be W_N is a torque divide the r_p in divide by cos phi.

So, once we know W_N, we should be able to find out the friction losses, the question comes, we are trying to find out this W_N through using this relation, but am I doing right job, is it any possibility the only one pair in contact and we know there will be some sort of clearance, if this gear tooth go in this spacing to avoid the jamming we keep some extra clearance; naturally that means when the one tooth pair engagement is over there will be some clearance and that is going to give a jerk.

Naturally, we need to design gear pair in a such a manner before losing first contact completely other pair should start contacting it, then only the smooth transition of the movement will occur, through smooth transition of the power will occur from one shaft to

other surface shaft, power can be transmitted when there are more than one pair of the teeth in contact.

May be the pitch point there may be only one pair in a contact but, away from the pitch point may be just a start or the just end of that naturally there need to be more than one or two pair minimum to be in a contact, so that we can get a transmission. When it happens naturally, this W_N calculation is not going to be right, because there will be some sort of load shearing and if there is a load sharing we should account that load shearing effect.

And what we say, we should use a contact ratio and contact ratio is generally more than one to be designed and more than one means that is a load sharing at the extreme points at the pitch point maybe same load, but at the extreme points there will be different load. And we need to evaluate the power loss, we need to find out the load at a different points, because the sliding velocity also going to change, so we if you want to find out the power loss at the different points, we require force at normal load at those points as well as, we require a sliding speed and that we require all this calculations.

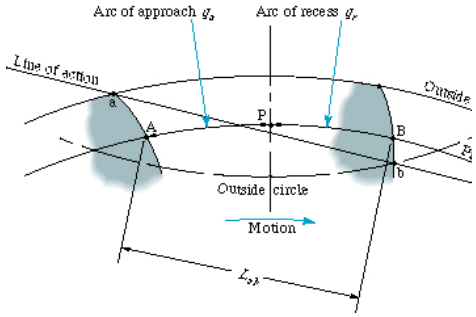
Now, here it is written contact ratio, so many questions marks, how to find out this contact ratio that may be the bigger expression, we will try to figure out and but, this value mostly first per gear sigma ranges from 1.4 to 1.7, so may be assume that lowest value should be 1.4 whenever we decide, it should be more than 1.4. When we talk about the helical gears the contact ratio is generally on a higher side that is 2.0, that is a reason why helical gears work smoothly compared to spur gear. The engagement is gradual and almost of the time the two pairs are sharing the load and that is an advantage.

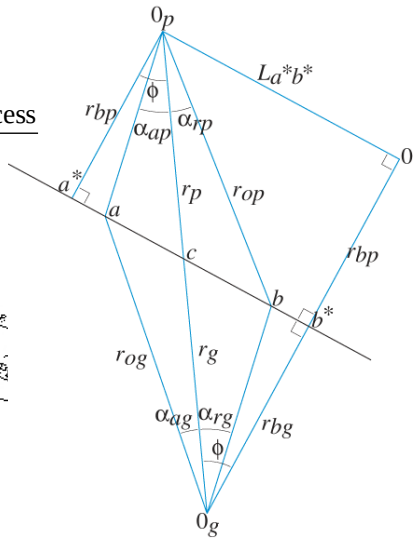
Because of this advantage many times people go ahead with spur gear geometry, because of a less complexity, but they try to make a high contact ratio gears they change profile to some manner, we say that they add addendum and dedendum. A larger addendum and dedendum, so that gear is modified to give us high contact ratio. Contact ratio is greater than 2.0 in those situations. And generally that gives a smooth operations, that gives us the lesser load per gear, gear teeth or gear tooth on the surface particularly for pinion and gear, now question comes how to figure out, how to estimate this contact ratio, so that can be done mathematically.

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Toothing

$$\frac{\text{Length of action}}{\text{Base_pitch}} = \frac{\text{Length of approach} + \text{Length of recess}}{(2\pi \frac{r_{bg}}{Z_g})}$$





For first, we can give some sort of description, so you assume this is one sketch and this is a one point, this is another point, we say the beginning of point and the end of the point, and this arc can be known as the angle of approach, when the gear teeth starts contacting, reaching to the pitch point and after that angle of recess, if they try to depart from that pair. So, there is a angle of approach, angle of recess, if we know this angle or we say can we can estimate what is a arc length. when they are coming into contact, and if we are able to estimate based on that we can figure out what will be the contact ratio.

Another word, we say contact ratio can be given as a length of action, the whole length of action and when they are coming in a mechanical contact assume that whatever the curve which we have shown over here, if we know the length and this is in polar coordinates, we can transfer to the retina coordinates, this curve if it is developed in a straight line, that will give me a length of action. And divide by the base pitch, that the base from where the gear teeth is started that goes, if that pitch is known to us we will be able to find out the contact ratio or we can say contact ratio is defined as a ratio of the length of action divided by base pitch; and this length of action can be divided in two parts, so length of approach and the length of recess and base pitch can be figured out by finding the circumference of pitch circle divided by number of teeth.

So, Z_g here is a number of teeth, r_{bg} is a base radius of the gear, so we are talking about the gear pair over here may be we can think about the pinion also but, here it is being used about gear f_g. So, radius of pitch circle is r_{vg} number of teeth on the gear is a Z_g, and we are talking about the circumference we say that 2π r_{bg} is a given circumference of the pitch circle divide

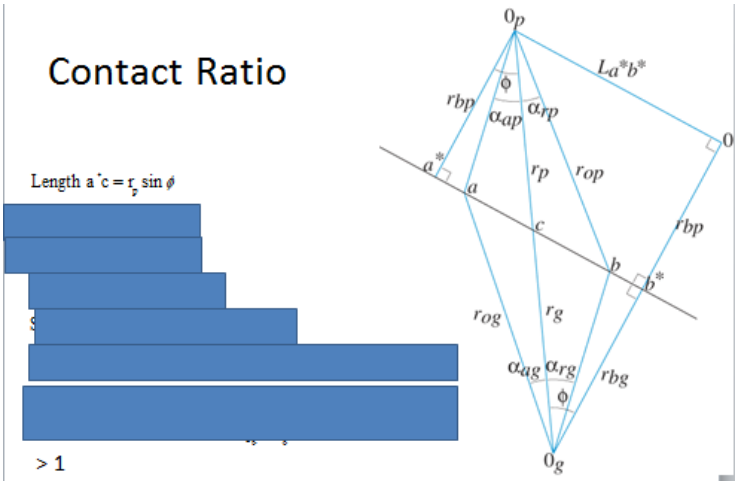
by number of gear teeth; that is going to give base pitch, and that once we know the base pitch, we know the length of the approach, length of recess, we can figure out what will be the contact ratio which is important to evaluate what will be the normal load on per tooth.

So, to be on surplus we can draw this line sketch say assume this o_p as a center of pinion o_g as a center of k, now point c is a pitch point contact, so there is a straight line in this case, we know length of action will be at some angle with this and this is a starting point I mean to say that a star is a point where the contact should start and b star is a point where the contact should end.

But, we know there will be lot of discontinuity, so what we do we provide some sort of flit at the start of contact and end of the contact, so the excess hysteresis are avoided, that is why in reality contact will start at the a, and end at the b, it is not a star, it is not a b star, so actual contact starts after the this base circle, it is not at the base circle point. We use some sort of flitting them, so this is a and end also, because of tooth we generally do not give very sharp edge, we give some sort of flit there, so tooth will be departing what is the separation will occur before a b star, and that is assuming that b, so this is a to b is length of action and we need to find out this length itself exactly.

We already know there will be 2π rbg divide by z_g, this is generally geometry parameter known to us before operation, but this action can be calculated based on the operating parameters, so we say we need to figure out what is the length of the approach, what is the length of recess and once we know that we will be able to find out what will be the contact ratio.

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Now, to find out the contact ratio we can start finding some lengths we say a star c, the length a star c can be figured out can be given in terms of r_p, we know this is a right angle triangle, and r_p is generally parameter known to us at the start particularly which from the manufacturing point of view, so r_p is if it is known and this is a sin phi, this is a phi we know the, this altitude angle, so we take that and on the try the a c can be given the a star c can be given in terms of r_p and sin phi that is given over here.

Now, we can figure out this length also a star b again there is a right angle triangle, a star b can be figured out using this relation, we can directly take this r_{op} also in terms of that, and we can find out this using this relations. So, this a r_{op} given this is a length of o_{pb}, that is a r_{op} is a outside diameter of the gear, if that is known and this is base radius is known to us by subtracting this we can figure out what will be the a*b, so that is why says the length a*b is a square root of r_{op} square minus r_{bp} square.

Now, what is our interest is a c length and c b length, we will first figure out what is a star c and second case we figure out what is a star b. Now, if we subtract this length from the complete length, naturally c b will be known to us, and that is given over here, the length c b is equal to a star b complete a star b minus a star c.

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Normal Load

$$\text{Length } a^*c = r_p \sin\phi$$

$$\text{Length } a^*b = \sqrt{r_{op}^2 - r_{bp}^2}$$

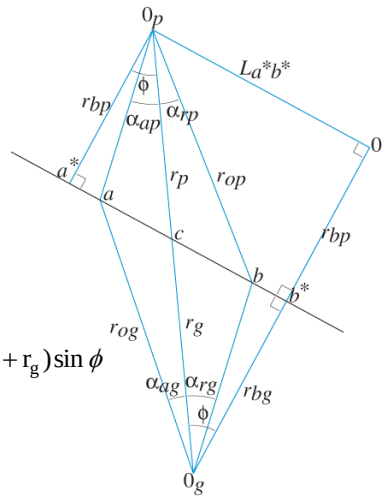
$$\text{Length } cb = a^*b - a^*c$$

$$\text{or, } cb = \sqrt{r_{op}^2 - r_{bp}^2} - r_p \sin\phi$$

$$\text{Similarly } ac = \sqrt{r_{og}^2 - r_{bg}^2} - r_g \sin\phi$$

$$\text{Length of action, } ab = \sqrt{r_{op}^2 - r_{bp}^2} + \sqrt{r_{og}^2 - r_{bg}^2} - (r_p + r_g)\sin\phi$$

$$\text{Contact ratio} = \frac{\sqrt{r_{op}^2 - r_{bp}^2} + \sqrt{r_{og}^2 - r_{bg}^2} - C\sin\phi}{2\pi r_{bg}/Z_g}$$



We can substitute this values, this is in mathematical term c b can be given as a square root of r_{op} square minus r_b square minus r_p, this is a pitch circle radius to sin phi, sin phi in the sin phi is known to us from a geometry point of view from manufacturing point of view r_p will be known to us from manufacturing point of view this also will be known, so that we will be able to figure out what will be c b.

Now, we do the reverse order we can figure out what will be the a c or we can start from here we can find out what will be the a c and that is the why we say this similarly, a c can be figured out from in terms of a r_{og} here what we figure out in terms of a pinion, here we are trying to figure out this value in terms of gear. So, rog square minus r_{bg} square both of the pinion is a outer diameter of, outer of diameter of gear this is a base diameter of gear. I will say radius in terms of radius this is a radius of outer radius of a gear and then base radius of gear minus again pitch circle diameter we are using, so pitch radius into sin phi assuming this the same pressure angle for the both the surfaces.

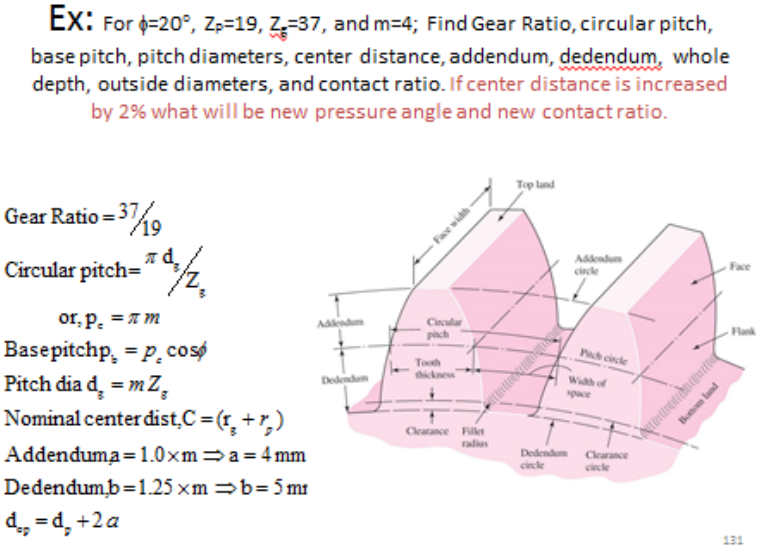
Using this what we say overall length will be cb plus ac and that is a what we say ab, the c is common and this will be turn out to be the first parameters related to the pinions second parameter related to gear, and this will be summation. Now, we can find out the contact ratio, so contact ratio can be given relation that is a numerator this is a denominator.

Once we know, what is the center distance c is a center distance the summation of rp plus r g that once we know this values, we can figure out what will be the contact ratio based on the contact ratio, the contact ratio is one naturally we know that the friction losses will be tremendous, but contact ratio is a lesser the friction losses will come down, because W_N will come down or will be reduced.

And it has been shown that contact ratio over the period of time will continuously decrease, initially when the gears are new contact ratio will be larger as it progresses or the wear out starts, then this contact ratio is going to decrease continuously that is why we say that spur gear, if I want 1.4 as a optimum value I should be able to keep it more slightly on a higher side, otherwise there will be some problem later may be say after 10000 hours or 15000 hours of operation, the gear will wear out and it start giving much lesser contact ratio that lesser contact ratio is going to reduce the stiffness of the system and the vibration will start and we say that there will be some sort of jack motion, there will be some sort of impact motion and the noise will start.

To illustrate this I have an example, I will just try to elaborate that and what this indication over here, contact ratio should be more than one always whenever we define.

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Now, just for the elaboration, I am just giving one example, how to use this parameter here the manufacturing data are given that is phi is equal to 20 degree, number of teeth on a pinion is 19 number of teeth on a gear are 37 module is 4, so we find gear ratio the simple Z_g divide by z_p, circular pitch to be figure out base pitch is diameter center distance over the addendum these are standard terms, addendum whole depth outside diameter and contact ratio.

In addition this is given, if the center distance is increased by 2 percent what will be new pressure angle new contact ratio, so we say contact ratio is given by this relation which was shown in previous slide, and gear which will symbol number of gear and number of teeth on gear divide by number of teeth on pinion circular pitch. This is the 2 pi r g divide by number of teeth and when we want to find out the base pitch, that it will be multiplied by cos phi that is why this P b can be correctly substitute over here.

Whenever we have a module and the number of teeth we can find out what will be the pitch diameter it is just simple multiplication, module into number of teeth is going to give us the pitch diameter, and that is what is been given the pitch diameter of the gear is m Z g.

Similarly, pitch diameter of pinion will be m into Z p and once we know the pitch diameter we can figure out what will be the radius, pitch radius, summation of pitch radius will be equal to center distance, and we are using the word nominal pitch center distance, nominal center distance reason being that, this values are hypothetical may change during assembling. Addendum and dedendum they are generally manufactured with and we say that standard thing is that there will be one m dimension above the pitch circle pitch radius, and below

distance between the base circle and pitch circle at anytime thickness between this two will be 1.25 times of module.

So, a is given, b is given to us we can figure out all the parameters and we can figure out what will be the contact ratio, now if we find the contact ratio it will turn out to be say 1.62, when the second part in this case is given that it is center distance increased by 2 percent, if a center distance is increased by 2 percent then the c is increased by 2 percent and that is being given the base pitch also will remain same, but r p will change and that will increase 2 percent, and that is going to give us a new angle, new pressure angle, so this new pressure angle will be slightly greater than 20 degree.

In my next lecture, we will continue with this example I will just repeat the slide and elaborate it and ensure some sort of the calculation with this kind of example, and will continue with friction and lubrication of the gears thank you, thanks for your attention.